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ANALYSIS OF THE MOTION OF TWO CLOSELY SPACED CYLINDERS

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ABSTRACT

The analysis of multi-bodies moving closely-spaced in a fluid region is complex. Analytical solutions can be found if the bodies are of simple forms and largely-spaced.

Under the assumptions of low amplitude motions and ideal fluid, the motion of two circular cylinders is analysed using the Boundary Element Method. If the motions are harmonic the added mass can easily be computed; these numerical results are presented in a graphical form for several configurations.

I. INTRODUCTION

The analysis of the flow around a group of bodies moving closely-spaced is very important in several fields; among them it is worth to mention the analysis of offshore structures and wave energy absorbers, the analysis of some parts of a nuclear power plant, etc.

For bodies of simple forms and largely spaced from each other it is possible to find analytical solution for the fluid dynamic problem. If the bodies have no simple forms or if they are closed-spaced, in general, one has to resort to numerical methods to get the solution of the fluid dynamic problem; these are the finite difference method (F.D.M.) and the finite element method (F.E.M.) |6|, which require the discretization of the fluid domain, and in addition presents extra complications when the fluid region is large or infinite |5|. However, as an alternative, one can use the boundary element method (B.E.M.) which requires only the discre

tization of the boundaries of the fluid domain.

This paper deals with the analysis of the flow around two closed spaced circular cylinders, moving harmonically in an infinite fluid region. The B.E.M. was used to solve the boundary value problem associated to the fluid flow. For the analysis of the problem the viscosity was neglected and the flow was supposed to be irrotational.

Added mass was calculated for different configurations and the numerical values show good agreement with those obtained using analytical methods.

II. FORMULATION OF THE PROBLEM

The equation of motion for an array of n bodies moving in a fluid region can be written as

$$|M| \{\ddot{n}\} + |C| \{\dot{n}\} + |K| \{\dot{n}\} = |F_H| \qquad (1)$$

where $|M|$ is the mass, $|C|$ is the structural damping, $|K|$ is the stiffness matrix and $|F_H|$ is the hydrodynamic force matrix.

The hydrodynamic force matrix can be split as

$$|F_H| = |N'| \{\ddot{n}\} + |C'| \{n\} + |K'| \{n\} + \{Q\} \qquad (2)$$

where $|M'|$ is the added mass, $|C'|$ is the fluid damping, $|K'|$ is the fluid elastic stiffness and $\{Q\}$ is the exciting force.

In this paper one is interested in the added mass for the simple case of two circular cylinder (bodies A and B) moving harmonically, with small amplitude, in an infinite fluid region- see figure (1). The body motion is normal to the x-axis and the viscosity is neglected. If the flow is irrotational there exists a velocity potential ϕ^*, the gradient of which is the velocity vector u^*.

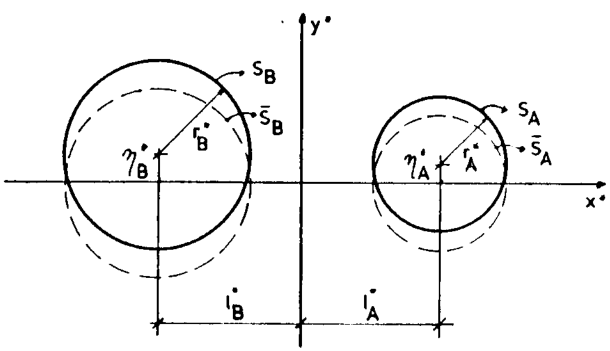

FIGURE 1 - Coordinate system for the motion of two cylinders

The pressure is related to the velocity through the Bernoulli equation

$$\frac{P^*}{\rho^*} + \frac{\partial \phi^*}{\partial t^*} + \frac{1}{2} \left|\left(\frac{\partial \phi^*}{\partial t^*}\right)^2 + \left(\frac{\partial \phi^*}{\partial y^*}\right)^2\right| = F(t^*) \qquad (3)$$

The velocity potential is obtained as the solution of the boundary value problem:

$$\nabla^2 \phi^* = 0 \qquad \text{in fluid region} \qquad (4)$$

$$\frac{DF_A^*}{Dt^*} = 0 \qquad \text{in } S_A \qquad (5)$$

$$\frac{DF_B^*}{Dt^*} = 0 \qquad \text{in } S_B \qquad (6)$$

$$|\nabla \phi^*| \to 0 \qquad \text{at large distances} \qquad (7)$$

where the surfaces S_A and S_B are defined respectively by

$$F_A = (x^* - \ell_A^*)^2 + (y^* - \eta_A^*)^2 + r_A^{*2} = 0$$

$$F_B = (x^* - \ell_B^*)^2 + (y^* - \eta_B^*)^2 + r_B^{*2} = 0$$

and the bodies motions by

$$\eta_A^* = \bar{\eta}_A^* \sin(\omega t^*) \qquad (8)$$

$$\eta_B^* = \bar{\eta}_B^* \sin(\omega t^* + \delta) \qquad (9)$$

Using $r^* = \max(r_A^*, r_B^*)$ and ω^{-1} as reference length and time, the boundary value problem is made nondimensional. In addition one writes the velocity potential as an assymptotic series

$$\phi(x, y, t) = \varepsilon\Psi_1 + \varepsilon^2\Psi_2 + \varepsilon^3\Psi_3 + \dots , \quad \varepsilon \to 0$$

where $O(\varepsilon) = O(\eta_A) = O(\eta_B)$ and the quantities without asterisk (*) are non-dimensional. Substituting this expansion in the equations (4) to (7) and retaining only the leading terms, results:

$$\nabla^2\Psi = 0 \qquad \text{in fluid region} \tag{10}$$

$$\frac{DF_A}{Dt} = 2y\eta_{A_t} = 2y\dot{\eta}_A \qquad \text{in the mean position } \bar{S}_A \tag{11}$$

$$\frac{DF_B}{Dt} = 2y\eta_{B_t} = 2y\dot{\eta}_B \qquad \text{in the mean position } \bar{S}_B \tag{12}$$

$$|\nabla\Psi| \to 0 \qquad \text{at large distances} \tag{13}$$

where $\frac{DF_A}{Dt}$ and $\frac{DF_B}{Dt}$ are the linearized form of the material derivative of the expressions defining the mean positions.

$$F_A = (x - \ell_A)^2 + y^2 - r_A^2 = 0$$

$$F_B = (x + \ell_B)^2 + y^2 - r_B^2 = 0 \ .$$

Note that in the equations above the subscript (1) in ψ_1 was deleted. It is also to note that a Taylor expansion was used to transfer the body condition from the actual to the mean position and that the time derivative does not change order of magnitude, that is $\omega = O(1)$; see reference |3|.

II.1 - Added Mass

The hydrodynamic loads on the bodies can be obtained through the relation |4|

$$\underset{\sim}{F}_H = - \int_C P \, \underset{\sim}{n} \, dS$$

where C is the body contour and $\underset{\sim}{n}$ is the unit external normal. To the order considered the pressure P is given by the time derivative of the velocity potential, that is

$$\underset{\sim}{F}_A = \int_{\bar{S}_A} \frac{\partial\Psi}{\partial t} \, \underset{\sim}{n} \, dS \tag{14}$$

$$\underset{\sim}{F}_B = \int_{\bar{S}_B} \frac{\partial\Psi}{\partial t} \, \underset{\sim}{n} \, dS \tag{15}$$

where $\underset{\sim}{F}_A$ and $\underset{\sim}{F}_B$ are the loads on bodies A and B.

In order to evaluate the above expressions it is convenient to use new velocity potentials Ψ_A and Ψ_B

$$\Psi = \dot{\eta}_A \Psi_A + \dot{\eta}_B \Psi_B \tag{16}$$

which are defined as shown in table below.

$\nabla^2\Psi = 0$	$\nabla^2\Psi_A = 0$	$\nabla^2\Psi_B = 0$
$\frac{DF_A}{Dt} = 2y\eta_{A_t}$	$\frac{DF_A}{Dt} = 2y$	$\frac{DF_A}{Dt} = 0$
$\frac{DF_B}{Dt} = 2y\eta_{B_t}$	$\frac{DF_B}{Dt} = 0$	$\frac{DF_B}{Dt} = 2y$
$\lvert\nabla\Psi\rvert \to 0$	$\lvert\nabla\Psi_A\rvert \to 0$	$\lvert\nabla\Psi_B\rvert \to 0$

Expressions (14) and (15) now take the form:

$$\underset{\sim}{F}_A = \underset{\sim}{F}_{AA} + \underset{\sim}{F}_{AB} = \ddot{\eta}_A \int_{\overline{S}_A} \frac{\partial \Psi_A}{\partial t} \underset{\sim}{n}\, dS + \ddot{\eta}_B \int_{\overline{S}_A} \frac{\partial \Psi_B}{\partial t} \underset{\sim}{n}\, dS \tag{17}$$

$$\underset{\sim}{F}_B = \underset{\sim}{F}_{BA} + \underset{\sim}{F}_{BB} = \ddot{\eta}_A \int_{\overline{S}_B} \frac{\partial \Psi_A}{\partial t} \underset{\sim}{n}\, dS + \ddot{\eta}_B \int_{\overline{S}_B} \frac{\partial \Psi_B}{\partial t} \underset{\sim}{n}\, dS\ ; \tag{18}$$

therefore the hydrodynamic loads (see expression (^{2})) are reduced to the hydrodynamic inertial forces: $\{F_H\} = |M'|\ \{\ddot{\eta}\}$.

The x and y components of the loads are

$$F_{AA_x} = \ddot{\eta}_A \left| \int_{\overline{S}_A} \frac{\partial \Psi_A}{\partial t} \underset{\sim}{n}\, dS \right|_x \qquad F_{AA_y} = \ddot{\eta}_A \left| \int_{\overline{S}_A} \frac{\partial \Psi_A}{\partial t} \underset{\sim}{n}\, dS \right|_y$$

$$F_{AB_x} = \ddot{\eta}_B \left| \int_{\overline{S}_A} \frac{\partial \Psi_B}{\partial t} \underset{\sim}{n}\, dS \right|_x \qquad F_{AB_y} = \ddot{\eta}_B \left| \int_{\overline{S}_A} \frac{\partial \Psi_B}{\partial t} \underset{\sim}{n}\, dS \right|_y$$

$\vdots \qquad\qquad \vdots$

where the parts between brackets on the right hand side of the above expressions represent the added mass. The added mass coefficients are defined |2| as shown in the examples below:

$$C_{AA_x} = \frac{M'_{AA_{yx}}}{\pi\, r_A^2} = \frac{1}{\pi\, r_A^2} \left| \int_{\overline{S}_A} \frac{\partial \Psi_A}{\partial t} \underset{\sim}{n}\, dS \right|_x$$

$$C_{AA_y} = \frac{M'_{AAyy}}{\pi\, r_A^2} = \frac{1}{\pi\, r_A^2} \left| \int_{\overline{S}_A} \frac{\partial \Psi_A}{\partial t} \underset{\sim}{n}\, dS \right|_y$$

$$C_{AB_x} = \frac{M'_{AB_{yx}}}{\pi (r_A + r_B)^2} = \frac{1}{\pi (r_A + r_B)^2} \left| \int_{\overline{S}_A} \frac{\partial \Psi_B}{\partial t} \underset{\sim}{n}\, dS \right|_x \tag{19}$$

$$C_{AB_y} = \frac{M'_{AB_{yy}}}{\pi (r_A + r_B)^2} = \frac{1}{\pi (r_A + r_B)^2} \left| \int_{\overline{S}_A} \frac{\partial \Psi_B}{\partial t} \underset{\sim}{n}\, dS \right|_y$$

III. SOLUTION

In order to get the solution of the boundary value problem use is made of the B.E.M., For that one start with the relation |1|

$$a_i \Psi(z_i) = \int_C \frac{\partial \Psi}{\partial n} \omega_i\, dC - \int_C \Psi \frac{\partial \omega_i}{\partial n}\, dC \tag{29}$$

where $a_i =$ 1 - if z_i is an interior point in the fluid region; 1/2 - if z_i is on the contour C and $\omega_i(Z) = \frac{1}{2\pi} \ell n \left(\frac{1}{|Z-Z_i|}\right)$

which represents a point source located at z_i, with unit intensity.

In the simplest case the boundary of the fluid region is divided in N straight segments whithin which Ψ is constant; therefore

$$\frac{1}{2} \Psi_i = \sum_{j=1}^{N} \frac{\partial \Psi_i}{\partial n} \int_{C_j} \omega_i \, dC - \sum_{j=1}^{N} \Psi_j \int_{C_j} \frac{\partial \omega_i}{\partial n} \, dC \qquad (30)$$

Defining

$$G_{ij} = \int_{C_j} \omega_i \, dC \quad \text{and} \quad \hat{H}_{ij} = \int_{C_j} \frac{\partial \omega_i}{\partial n} \, dS \qquad (31)$$

one has.

$$\frac{1}{2} \Psi_i = \sum_{j=1}^{N} \frac{\partial \Psi_j}{\partial n} \cdot G_{ij} - \sum_{j=1}^{N} \Psi_j \hat{H}_{ij}$$

and if

$$H_{ij} = \begin{cases} \hat{H}_{ij} , & i \neq j \\ \hat{H}_{ij} + \frac{1}{2} , & i = j \end{cases} \qquad (32)$$

one is left with

$$|H_{ij}| \{\Psi_j\} = |G_{ij}| \{\frac{\partial \Psi_i}{\partial n}\} \qquad (33)$$

or

$$|A_{ij}| \{x_j\} = \{b_i\}$$

where

$$A_{ij} = H_{ij}$$

$$x_j = \Psi_j$$

$$b_i = \sum_{j=1}^{N} G_{ij} \frac{\partial \Psi_i}{\partial n}$$

when $\frac{\partial \Psi}{\partial n}$ is a known function on C.

III.1 - Numerical Results

For the problem under consideration expression (29) takes the form:

$$a_i \Psi_i = \int_{\bar{S}_A+\bar{S}_B} \left| \frac{\partial \Psi}{\partial n} \omega_i - \Psi \frac{\partial \omega_i}{\partial n} \right| dS + \int_{S_\infty} \left| \frac{\partial \Psi}{\partial n} \omega_i - \Psi \frac{\partial \omega_i}{\partial n} \right| dS$$

where S_∞ means the boundary at large distances and the integral there vanishes |3|. The integrals on $\bar{S}_A + \bar{S}_B$ are easily evaluated when Ψ is constant on each element |3|.

Each surface, $\bar{S}_A$ and $\bar{S}_B$, were devided into 40 elements as shown in Figure (2). When 80 elements were used the numerical results didn't show significant changes.

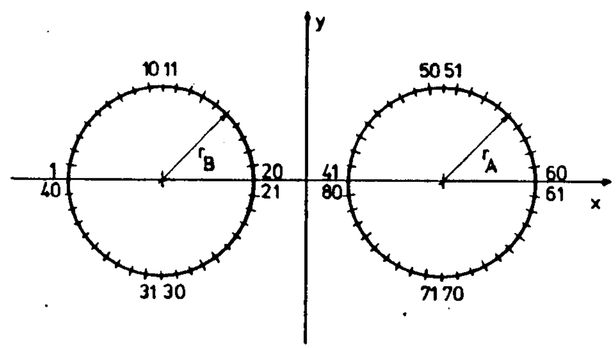

FIGURE 2 - Discretization model

Figure (3) shows the coefficients C_{AA_y} and C_{BB_y} for three values of $R = r_A/r_B$ as a function of the gap between the cylinders, which is given by the parameter $L = 1 - |(r_A + r_B)/(\ell_A + \ell_B)|$. These results show good agreement with those analytically obtained |3|. Note however that for very small gaps the solution looses its meaning since viscous effects can not be neglected.

Figure (4) shows the coefficients C_{AB_y} and C_{BA_y} again plotted against L. These coefficients represent the influence on the added mass of a cylinder due to the presence of the other one, while C_{AA_y} and C_{BB_y} represent the added mass of each cylinder in the absence of the other.

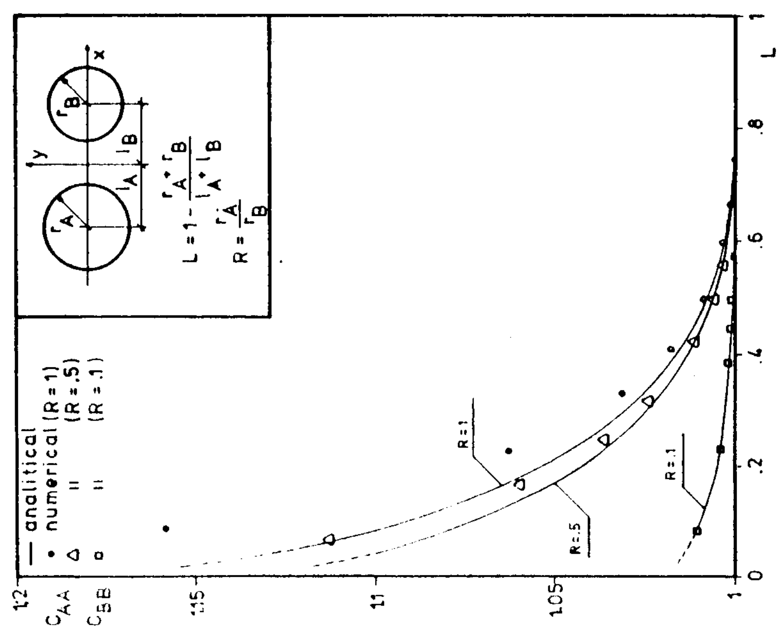

FIGURE 3 - Coefficients C_{AA} and C_{BB}

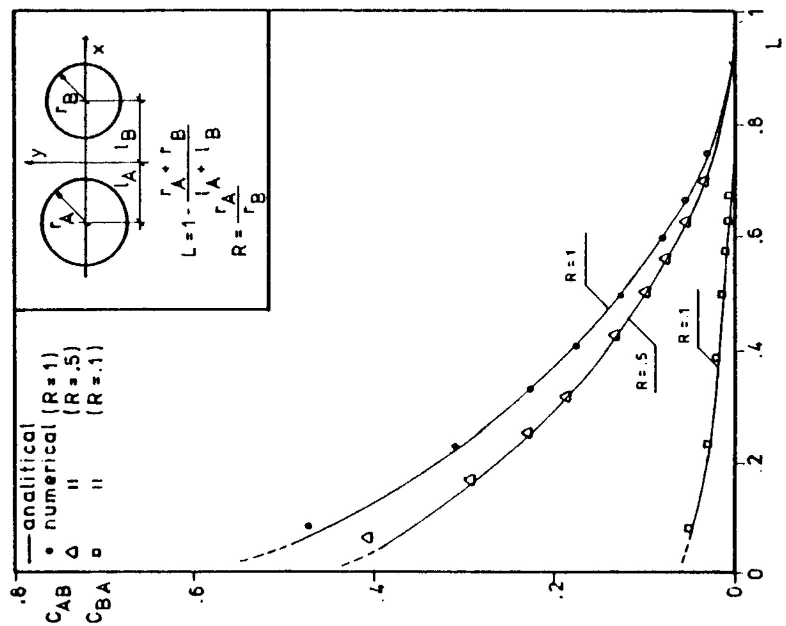

FIGURE 4 - Coefficients C_{AB} and C_{BA}

The C_{AA_x}, C_{BB_x}, C_{AB_x}, ... are not present since the motion is only in the y direction. The attaction forces on the cylinders are of higher order since they arise due to the quadratic terms in Bernoulli equation.

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