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STABILITY OF INTACT SHIPS IN WAVES

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ABSTRACT

The stability of ships is examined as a problem of dynamics. The motions of the ship in waves are analysed and the implications of perturbations to these motions are investigated. This analysis is compared with the classical approach of linear dynamics.

The phenomenon of parametric excitation is discussed for the heave, roll and pitch motions. Stability calculations are presented for a stern trawler.

1 - INTRODUCTION

In 1962 the Intergovernmental Maritime Consultative Organization (IMCO) established a comittee to investigate the stability problem of all types of ships, including fishing vessels. This comittee completed its work in 1968 and produced its basic stability criteria, based on an extensive survey of various national stability regulations and on the statistical casualty records.

These criteria are not always satisfactory, as in the case of several casualties of vessels (especially small vessels) which have satisfied the IMCO criteria, sometimes with an ample margin, but capsized in adverse weather conditions. Examples of these casualties are to be found in the papers by Morrall |1, 2|, Dahle and Kjaerland |3|, Kure and Bang |4|. In 1975, during the 1^{st} International Conference on Stability of Ships and Ocean Vehicles, Kuo and Gordon |5| carried out a survey on the opinions of the delegates on the stability problem. It became apparent that many delegates consider the IMCO criteria as unreliable and insufficient.

It may not be surprising, perhaps, that these criteria are unreliable and insufficient if one has in mind that these criteria are essentially based on hydrostatics. A large proportion of the operating life of a ship is spent in the presence of surface waves, and occasionally encountering very severe conditions. As a matter of fact, the capsizing of an intact ship is a phenomenon which by its very nature involves excessive motions in the presence of sea waves. It is clearly a matter of dynamics.

The approach to be used in a truly dynamic analysis must be to set up suitable differential equations of motion for a rigid ship in waves, and then, to examine the stability of the possible solutions. This is, in essence, the procedure when studying the directional stability of ships in calm seas. Linear equations of manouvering are set up, usually employing slow-motion derivatives, and stability conditions are then deduced by means of the Routh-Hurwitz conditions |6|. It is necessary now to investigate how these conditions might be modified when the effects of sea waves are introduced in the analysis.

In this paper, the implications of wave effects on ship stability are discussed, specially having regard to resonances. Nonlinearities are considered and these are show to be relevant in identifying new resonant conditions. In this context, the phenomenom of parametric excitation is discussed and some of its features are elucidated.

2 - LINEAR ANALYSIS

2.1 - Linear Equations of Motion

When linear equations are employed to describe the motions at sea of a rigid vessel in its 6 degrees of freedom, two separate sets of equations corresponding to symmetric and antisymmetric motions are obtained |7|. The equations for symmetric motions are

$$\left.\begin{aligned} \Delta X - mg\theta &= m\dot{u} \\ \Delta Z &= m\,(\dot{w} - q\bar{U}) \\ \Delta M &= I_y\,\dot{q} \end{aligned}\right\} \qquad (1)$$

whereas the equations for antisymmetric motions are

$$\left.\begin{aligned} \Delta Y + mg\phi &= m\,(\dot{v} + r\bar{U}) \\ \Delta K &= I_x\dot{p} - I_{xz}\,\dot{r} \\ \Delta N &= I_z\dot{r} - I_{xz}\,\dot{p} \end{aligned}\right\} \qquad (2)$$

These equations are valid for axis at the center of gravity C of the ship, for ships with forward speed $\bar{U}$, mass m, inertia properties I_x, I_y, I_z and product of inertia I_{xz}. See Figure 1. For small departures from the reference motion, the corresponding external forces are ΔX, ΔY and ΔZ, whereas the external moments are ΔK, ΔM and ΔN. The perturbed linear motions are denoted by u, v and w representing surge, sway and heave motions whereas the angular displacements are ϕ, θ and Ψ, with p, q and r representing roll, pitch and yaw angular velocities. Dots are used to indicate time derivations and g is gravity.

Considering that the external forces and moments ΔX, ΔY, ΔZ, ΔK, ΔM and ΔN may be represented by Taylor series, both sets of equations may be expressed in the following matricial form:

$$\underset{\sim}{A}\ddot{\underset{\sim}{q}} + \underset{\sim}{B}\dot{\underset{\sim}{q}} + \underset{\sim}{C}\underset{\sim}{q} = \underset{\sim}{Q}(t) \qquad (3)$$

where, for example, in the case of antisymmetric motions, the matrices are |7|

$$\underset{\sim}{A} = \begin{bmatrix} m - Y_{\dot{v}} & - Y_{\dot{p}} & - Y_{\dot{r}} \\ -K_{\dot{v}} & I_x - K_{\dot{p}} & -(I_{xz} + K_{\dot{r}}) \\ -N_{\dot{v}} & -(I_{xz} + N_{\dot{p}}) & I_z - N_{\dot{r}} \end{bmatrix},$$

$$\underset{\sim}{B} = \begin{bmatrix} - Y_v & - Y_p & m\bar{U} - Y_r \\ - K_v & - K_p & - K_r \\ - N_v & - N_p & - N_r \end{bmatrix} , \qquad \underset{\sim}{C} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta.\overline{GM} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where $\Delta = mg$ is the ship displacement and $\overline{GM}$ is the metacentric height. In this case the displacement vector is

$$\underset{\sim}{q} = \begin{bmatrix} \int v dt \\ \phi \\ \psi \end{bmatrix}$$

and in a sea of regular waves of frequency ω and angle of incidence χ, the exciting vector $\underset{\sim}{Q}(t)$ may be expressed as

$$\underset{\sim}{Q}(t) = \begin{bmatrix} Y \\ K \\ N \end{bmatrix} e^{i\omega_e t}$$

where

$$\omega_e = \omega - \frac{\bar{U}\,\omega^2 \cos\chi}{g}$$

is the frequency of encounter.

2.2 - Resonant Conditions

The resonant conditions identifiable by the two sets of equations are those where the encounter frequency coincides with one of the system's natural frequencies, viz, the heave, roll and pitch natural frequencies. The remaining modes are in the horizontal plane, unopposed by restoring forces or moments, their responses being nonresonant in nature. Furthermore, in the context of systems described by equations (1) and (3), the heave and pitch modes are uncoupled from the roll mode.

The response at resonance depends on the magnitude of the damping forces and moments and the degree of tuning. The damping of pitch and heave motions is generally subcritical, but sufficient to prevent highly tuned resonant responses. For the rolling mode the static restoring moment is small for conventional ships, while the damping is weak. Thus, resonant motions in roll occur with a large amplitude. Therefore, it would appear that from the ship capsize point of view only the roll resonance is of importance, the heave and pitch resonant conditions being important only for "seaworthiness" assessments of ships. However, it is accepted that most ships can withstand, despite heavy motions, this resonant condition in roll; this is naturally a dangerous situation but which cannot account on its own for many of the capsizes |8|.

2.3 - Stability Analysis

The stability analysis of a linear matrix equation like equation (3) is a well developed field of the theory of linear differential equations. Bryan (1911) |9| in the context of aerodynamics and Kunzel and Weinblum (1938) |10| in the study of ship motions made use of these methods. It is essentially a problem of determining the eigenvalues and eigenvectors of the matrix equation

$$\underset{\sim}{A}\ddot{\underset{\sim}{q}} + \underset{\sim}{B}\dot{\underset{\sim}{q}} + \underset{\sim}{C}\underset{\sim}{q} = \underset{\sim}{Q}(t) \qquad (3)$$

In many problems the three equations in each set are coupled together very weakly, so that the analysis may be simplified further to that of solving two simultaneous linear differential equations. The surge mode is uncoupled from the heave and pitch modes in the symmetric motions and roll is uncoupled from the sway and yaw modes in the antisymmetric motions.

In practice there is no possibility of instabilities occurring in heave and pitch motions. As stated before, these modes are more direcly related to "seaworthiness" assessments of ships rather than to the study of the stability of ships, in the sense

that an instability would imply in a progressive build-up of a motion, and that, of course, is not a feature of the heave and pitch behaviour of ships.

The analysis of the sway and yaw motions is commonly denoted directional stability analysis. It is usual to find in this analysis that a particular ship would undergo a growth of her motion, meaning that the ship would be directionally unstable. The directional stability of the system is determined by a property of the roots of the characteristic equation of the differential equations of motion. Indeed, it is not even necessary to calculate the roots of the characteristic equation: directional stability may be assessed by means of the so-called Routh-Hurwitz criteria |11|.

According to these criteria, when sway and yaw motions are taken together, directional stability would prevail for |6|:

$$Y_v N_{\dot{r}} + N_v (m\bar{U} - Y_r) > 0. \qquad (4)$$

This is the usual form of the directional stability criterion for flat calm water and it will be seen to contain no term relevant to rolling ϕ or to a rolling moment K.

When roll is considered independently of sway and yaw, the stability criterion is merely the condition that the metacentric height of a ship in roll $(\overline{GM})$ is positive, or

$$K_\phi < 0 . \qquad (5)$$

It should be remembered that the derivative K_ϕ may vary with the ship's forward speed $\bar{U}$. If this variation is small, however, it can be deduced that a ship which is statically stable in roll is also dynamically stable.

Clearly, when linear equations of sway, roll and yaw are taken together the coupling between these linear equations will determine a stability criterion which is merely a composition of the stability requirements of "directional stability" in planar motion and "righting stability" in roll |12, 13|:

$$\Delta.\overline{GM}\ [Y_v\ N_r + N_v\ (m\bar{U} - Y_r)] > 0. \qquad (6)$$

3 - COMMENTS ON THE USE OF NON-LINEAR TERMS

The linear analysis of the previous sections shows that only two dangerous situations may be identified within the context of linear theory. These are (a) resonance in roll; (b) possibility of directional instability.

As discussed previously, (a) is a known situation in practice when heavy motions can ocour, but other causes must be found to explain why so many capsizings still occur nowadays. On the other hand, the possibility of directional instability in a vessel cannot on its own be responsible for a ship capsizing. The phenomenon of broaching is, of course, related to directional instability, but again it may be argued that few capsizes are caused in the sequence of a broach-to |14, 15|.

Therefore, any effort to get a better understanding of the complex problem of ship capsize must attempt to add new terms to the equations of motion, in the hope that the more complex mathematical model will be capable of identifying phenomena not identified by the linear analysis. In other words, some form of non-linearity must be introduced. A considerable amount of work has been invested in the study of non-linearities in the roll equation |4, 16, 17, 18| but usually with the main purpose of describing inclinations assumed to be large. The success of these works cannot be said to be encouraging, due to the known complexities of non-linear theory.

In the following sections, some non-linearities will be introduced into the equations, with the sole purpose of identifying dangerous situations for the ship; strictly speaking, with the aim of specifying new resonant conditions. The known technique of reducing a non-linear equation to its variational equation will be employed to reduce the system to a linear form amenable to stability analysis. The advantages of this technique in the context of ship stability will be stressed later.

4 - MATHIEU EQUATION

4.1 - The Influence of Heave and Pitch on the Roll Motion

The metacentric radius of a ship, $\overline{BM}$, is defined as the ratio of the waterplane moment of inertia to the immersed volume ∇. The metacentric height, $\overline{GM}$, used as a measure of stability, is defined by

$$\overline{GM} = \frac{I_T}{\nabla} + \overline{KB} - \overline{KG}$$

where $\overline{KB}$ is the vertical coordinate of the centre of buoyancy and $\overline{KG}$ is the vertical coordinate of the centre of gravity, as illustrated in Figure 2.

If the ship is assumed to be heaving and pitching in a seaway, the continuous modification of the wetted portion of the hull will produce changes in $\overline{BM}$ and $\overline{KB}$, thus inducing a continuous modification of $\overline{GM}$. If the heave and pitch motions are assumed to be harmonic the $\overline{GM}$ variation will then be periodic with a frequency equal to the encounter frequency ω_e |15|.

If, in equation (3), the roll mode is uncoupled from the others and if it is assumed that the roll restoring moment varies periodically with frequency ω_e, the roll equation becomes

$$(I_x - K_{\dot{p}})\ddot{\phi} - K_p\dot{\phi} + \rho g\nabla\left[\overline{GM} + \delta\overline{GM}\cos(\omega_e t - \rho_\omega)\right]\phi = K(t) \qquad (7)$$

where $\delta\overline{GM}$ is the amplitude of $\overline{GM}$ variation, and ρ_ω is a phase angle. The value of $\delta\overline{GM}$ is a function not only of the geometric properties of the hull, but of the heave and pitch amplitudes as well. This $\overline{GM}$ variation is expected to be small. Actually, this harmonic term is of second order, non-existent in the linear (first order) derivation of the equation. Nevertheless, equation (7) is linear; it is the homogeneous version of equation (7) that informs whether or not the complete solution of this equation can attain its steady state form. Thus, taking equation (7) in its homogeneous form and making the transformations

$$\phi(t) = x(t) \, . \, e^{\frac{K_p}{2\,(I_x - K_{\dot{p}})} t} \tag{7a}$$

$$\tau = \frac{\omega_e t - \rho_\omega}{2} , \tag{7b}$$

the following equation is obtained:

$$\frac{d^2x}{d\tau^2} + (\sigma^2 + \varepsilon \cos 2\tau)\, x = 0 \tag{8}$$

where

$$\sigma^2 = 4 \left[\left(\frac{\omega_4}{\omega_e}\right)^2 - \left(\frac{-K_p}{2\omega_e\,(I_x - K_{\dot{p}})}\right)^2 \right] \tag{8a}$$

$$\varepsilon = \frac{4\,\rho\, g\, \nabla \,.\, \delta\, \overline{GM}}{\omega_e^{\,2}\,(I_x - K_{\dot{p}})} \tag{8b}$$

and ω_4 is the roll natural frequency given by

$$\omega_4 = \sqrt{\frac{\rho\, g\, \nabla\, \overline{GM}}{I_x - K_{\dot{p}}}} \tag{8c}$$

Equation (8) is denominated Mathieu equation. Although its general closed solution is not known, the stability of its solutions may be deduced from a stability chart in the (ε,τ) - plane. An illustration of this chart is given in Figure 3 showing the zones of stability and instability, where the former are thatched. For small values of ε, instability occurs for

$$\sigma = m \qquad , \qquad m = 1, 2 \ldots \tag{9}$$

and this means that a system like the one described by equation (8) displays a whole sequence of resonant conditions. This phenomenon has been denoted as a case of resonance between the "free oscillations" of the solutions of the equation

$$\ddot{x} + m^2 x = 0$$

of frequency m and the periodic disturbance $\varepsilon \cos 2t$ |19|.

It should be pointed out that the Mathieu equation (8) describes the behaviour of a system with no damping. The roll damping is considered in expression (7a) from which the behaviour of the roll variable $\phi(t)$ can be investigated. If only for a moment the roll damping is disregarded, it is found that instabilities may occur for small ε close to the values

$$\frac{\omega_4}{\omega_e} = \frac{m}{2} \quad , \quad m = 1, 2, \ldots \tag{10}$$

The rate of increase of the amplitude on an unstable solution depends above all, on the amplitude of the parametric excitation ε. Comparing the zones of instabilities in Figure 3 it can be noted that the first is the widest one and can be reached with small values of parametric excitation. These two aspects make the first region of instability the most dangerous one.

4.2 - Effect of Damping

Consider now the effect of roll damping. If parametric instability may occur in the neighbourhood of small ε values and for conditions given by equation (9) the roll angle ϕ may or may not become unstable in these cases. The transformation

$$\phi(t) = x(t) \cdot e^{\frac{K_p}{2\,(I_x - K_{\dot{p}})}\,t} \tag{7a}$$

indicates, for $K_p < 0$, that provided the exponential in equation (7a) converges faster than $x(t)$ at the mentioned ratios of frequencies the roll motion will be stable in the mathematical sense but most likely will be large for small damping coefficients. The implication in the case of a ship may be a capsize facilitated by these large motions. Whether the exponential in equation (7a) converges faster than $x(t)$ at a particular frequency is defined by a "threshold" value of the damping coefficient dependent on the

$\overline{GM}$ variation |20|. Yet, in any case, even for values of damping coefficients larger than the "threshold" value, large roll motions are likely to occur at the resonant frequencies before the damping action is felt to attenuate the motion.

The importance of the damping in the stability analysis of an equation reduceable to the Mathieu equation is examined by Bolotin |21|. The "threshold" values are discussed and it is shown that the regions of instability of equation (7) lie inside the regions of instability of equation (8).

It is accepted that the instabilities discussed above are instabilities occuring in a linear equation, and as the rolling angle increases the equation will become invalid |22|. Nevertheless, the sort of information yielded by this equation is important having regard to the possibility of defining "dangerous frequencies": more exciting frequencies may now be identified as potentially dangerous (mainly the one related to the first region of instability) to the ship.

5 - USE OF THE VARIATIONAL EQUATION

It becomes clear from the linear analysis of Section 2 that wave effects do not alter the stability results derived from the classical calm water condition of manoeuvrability studies. This is essentially what equation (6) expresses. It may be revealing to consider this point more closely. What equation (6) indicates is that, when considered to the first order, the equation of motion will give information only about perturbations superimposed to a reference motion which in this case is a straight course with speed $\bar{U}$. In other words, the linear stability analysis will indicate whether or not perturbations to a straight course may or may not subside. When the motions of a ship in waves of small amplitudes are considered, these will appear with the same order of magnitude as the perturbations considered in the manoeuvring equations.

Hence, it is necessary to increase the order of magnitude of the equation of motion, in order to consider perturbations to the oscillatory motion of a ship in waves. This can be done in a mathematically consistent way by obtaining the variational equation of the non-linear system, a well known technique of non-linear differential equations which in discussed in Appendix 1. Non-linear equations which have their linear variational equations stable are denoted in applied mathematics as being "infinitesimally stable" |19|, or "stable to the first approximation" |11|.

Considering non-linear pure roll, the non-linear terms being of hydrostatic origin, the equation may be written as

$$(I_x - K_{\dot p})\ddot\phi - K_p\dot\phi - K_\phi\phi - K_{\phi\phi}\phi^2 - K_{\phi\phi\phi}\phi^3 - \ldots = K(t)$$

and by symmetry considerations

$$(I_x - K_{\dot p})\ddot\phi - K_p\dot\phi + \rho\, g\, \nabla\, \overline{GM}\,(\phi + C_3\phi^3 + C_5\phi^5 + \ldots) = K(t) \qquad (11)$$

where C_3, C_5 ... are constants representing terms of the complete curve of restoring arm of a ship, Figure 4.

Without loss of generality the terms proportional to ϕ^5 and higher orders will be disregarded. Now, by assuming the solution to be of the form

$$\phi(t) = \Psi(t) + x(t) \qquad (12)$$

where. $\Psi(t)$ is a steady state solution and $x(t)$ is a perturbation, the linear variational equation may be obtained by assuming

$$\Psi(t) = \Psi_o \cos(\omega t - \tau_4) \qquad (13)$$

and operating as described in the appendix. The variational equation thus becomes

$$(I_x - K_{\dot p})\ddot x - K_p\dot x + \rho\, g\, \nabla\, \overline{GM}\,\{A + B\cos 2(\omega t - \tau_4)\}\, x = 0 \qquad (14)$$

where

$$B = \frac{3\, C_3\, \Psi_o^{\,2}}{2} \quad ; \qquad A = 1 + B$$

It can be easily seen that by suitable transformations the above equation may be expressed in the canonical form of the Mathieu equation (8). Thus it is seen that the variational equation of the non-linear roll equation is in the form of a Mathieu equation, discussed in section 4, which is amenable to a stability analysis. It should be pointed out that equation (7) was deduced following a somewhat heuristia procedure, while equation (14) is mathematically consistent and offers, thus,a firm basis for the study of ship stability.

As discussed previously, equation (14) may display instabilities, especially at certain frequencies. As an illustration of the parametric instability, Figure 5 shows the time-history solution of equation (14) for the case of a transom-stern trawler |13|. Figure 5a shows a case of parametric resonance with instability whereas Figure 5b shows a damped oscillation for the same level of parametric excitation. It is clearly seen from the figure that parametric tuning is very important in the development of large motions. The motion displayed in Figure 5a is an instability super-imposed to the oscillatory motion of the stern trawler in waves and this growing instability may mean a capsize of the vessel in less than three cycles.

6 - CONCLUSIONS

The real nature of the intact ship stability problem is still not very well understood. Traditional techniques of predicting righting stability are based on hydrostatics. This is, perhaps, the very reason why these techniques have proved to be inadequate in some circumstances. As a matter of fact the stability problem of a ship is a dynamic problem, and as such, it is discussed here.

In the first part of this paper, the adequacy of the linear stability assessment is discussed. The equations of motion are taken in their coupled form. The significance of the couplings is investigated having regard to resonance and instabilities. It is shown that to the first order, the traditional calm water stability analysis is valid when wave effects are added to the problem.

In the second part of the paper, the role of non-linear ship dynamics is discussed. The method adopted here is one that attempts to avoid unnecessarily complicated mathematics: non-linear terms are introduced but the equations are reduced to their linear variational form. It is demonstrated, in a mathematically consistent way, how higher order terms in the roll equation are associated with parametric excitation terms in the Mathieu equation.

The method is illustrated solely in terms of the roll equation. There can surely be no doubt that it must be investigated much more fully particularly in respect of:

- coupled motions
- rudder operation

- random seas
- mooring systems.

The study of the intact stability of ships in waves is certainly a large and somewhat specialized undertaking, but it can reasonably be concluded that the mathematical model outlined here has much to offer. More resonant conditions are revealed now and these certainly constitute dangerous situations for ships at sea. How dangerous these resonant conditions may be is a matter for experimental studies.

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APPENDIX 1

VARIATIONAL EQUATION

In this appendix it will be shown how the variational equation to the first approximation may be obtained.

Sets of n second order differential equations of the form

$$\ddot{\tilde{q}} + \tilde{B}_1\dot{\tilde{q}} + \tilde{C}_1\tilde{q} = \tilde{Q}_1(t) \tag{A-1}$$

may be reduced to a set of 2n first order differential equations (see Stoker |23|).

Let

$$\dot{q}_1 = \beta_1 \;, \quad \dot{q}_2 = \beta_2 \;, \quad \ldots, \quad \dot{q}_n = \beta_n$$

$$q_1 = \beta_{n+1} \;, \quad q_2 = \beta_{n+2} \;, \quad \ldots, \quad q_n = \beta_{2n}$$

and

$$\dot{\beta}_{n+1} = \beta_1 \;, \quad \dot{\beta}_{n+2} = \beta_2 \;, \quad \ldots, \quad \dot{\beta}_{2n} = \beta_n$$

Equation (A-1) can then be rewritten as

$$\dot{\tilde{\beta}} + \tilde{M}\tilde{\beta} = \tilde{Q}(t) \tag{A-2}$$

where

$$\underset{\sim}{M} = \left[\begin{array}{c|c} \underset{\sim}{B}_1 & \underset{\sim}{C}_1 \\ \hline -\underset{\sim}{I} & \underset{\sim}{0} \end{array}\right] \quad \text{and} \quad \underset{\sim}{Q}(t) = \left[\begin{array}{c} \underset{\sim}{Q}_1(t) \\ \hline \underset{\sim}{0} \end{array}\right].$$

Another way of writing equations (A-2) is:

$$\dot{\beta}_i = f_i(\beta_1, \beta_2, \ldots, \beta_{2n}; t) \quad , \quad i = 1, \ldots, 2n \qquad \text{(A-3)}$$

Assume now that a steady state solution exists and is the vector $\alpha(t)$. The real motion will deviate from the steady state motion and the components of real motion are denoted as

$$\alpha_1(t) + \xi_1(t);\ \alpha_2(t) + \xi_2(t);\ \ldots\ ;\ \alpha_{2n}(t) + \xi_{2n}(t)$$

Substitution of $\alpha_i(t) + \xi_i(t)$, $i = 1, \ldots, 2n$ into the equations of motion yield the following set of equations:

$$\dot{\alpha}_i + \dot{\xi}_i = f_i(\alpha_1, \alpha_2, \ldots, \alpha_{2n}; t) + \sum_{k=1}^{2n} \left[\left(\frac{\partial f_i}{\partial \beta_k}\right)_{\beta_k = \alpha_k(t)} \cdot \xi_k + O(\xi_k) \right] \qquad \text{(A-4)}$$

and since $\alpha(t)$ is solution to (A-3) it follows that

$$\dot{\xi}_i = \sum_{k=1}^{2n} \left[\left(\frac{\partial f_i}{\partial \beta_k}\right)_{\beta_k = \alpha_k(t)} \cdot \xi_k + O(\xi_k) \right], \quad i = 1, \ldots, 2n \qquad \text{(A-5)}$$

where the derivatives in (A-5) are evaluated at $\beta_k = \alpha_k(t)$ and

$$\left[O(\xi_k)/\xi_k\right] \rightarrow 0 \quad \text{as} \quad \xi_k \rightarrow 0$$

Thus it is possible to reduce the problem of the stability of the solutions of (A-3) (or (A-1)) to the problem of the stability of the trivial solution of equation (A-5).

The linear system

$$\dot{\xi}_i = \sum_{k=1}^{2n} \left(\frac{\partial f_i}{\partial \beta_k}\right)_{\beta_k = \alpha_k(t)} \cdot \xi_k \quad , \quad i = 1, \ldots, 2n$$

is called the "linear variational system" relative to system (A-3). Systems which have their linear variational systems stable are usually denoted in applied mathematics as being "infinitesimally stable" or "stable to the first approximation".

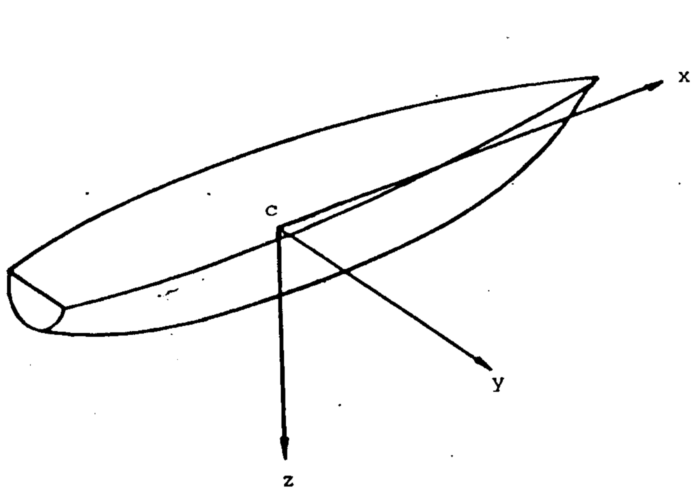


Figure 1 - System of axes with origin at centre of gravity C of rigid ship.

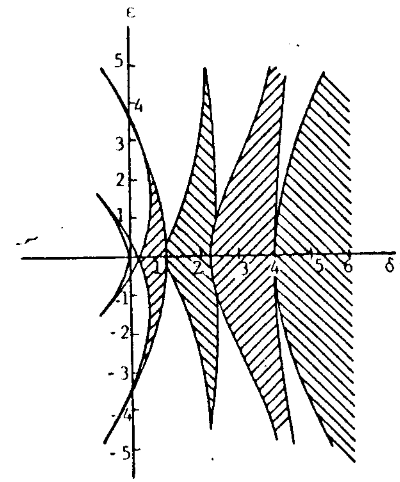
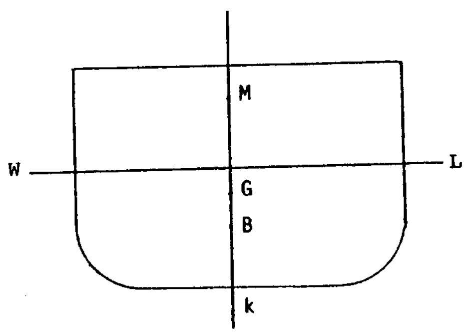


Figure 3 - Stable and unstable regions for the Mathieu equation.



$$\overline{GM} = \overline{BM} + \overline{KB} - \overline{KG}$$

$$\overline{BM} = \frac{I_T}{\nabla}$$

Figure 2 - Parameters defining initial stability.

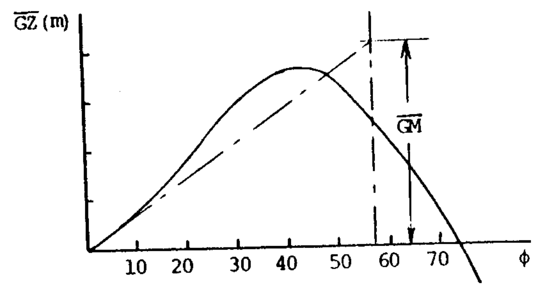


Figure 4 - Curve of righting arm.

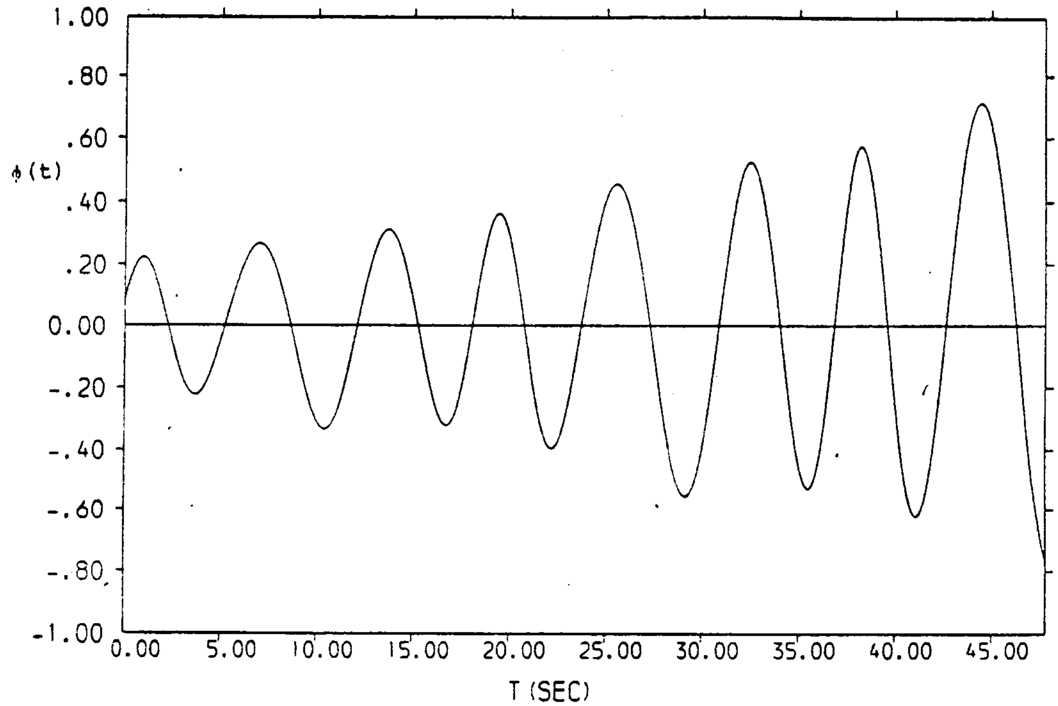


Figure 5a - Roll motion for transon stern trawler. Unstable motion for exciting frequency close to twice roll natural frequency.

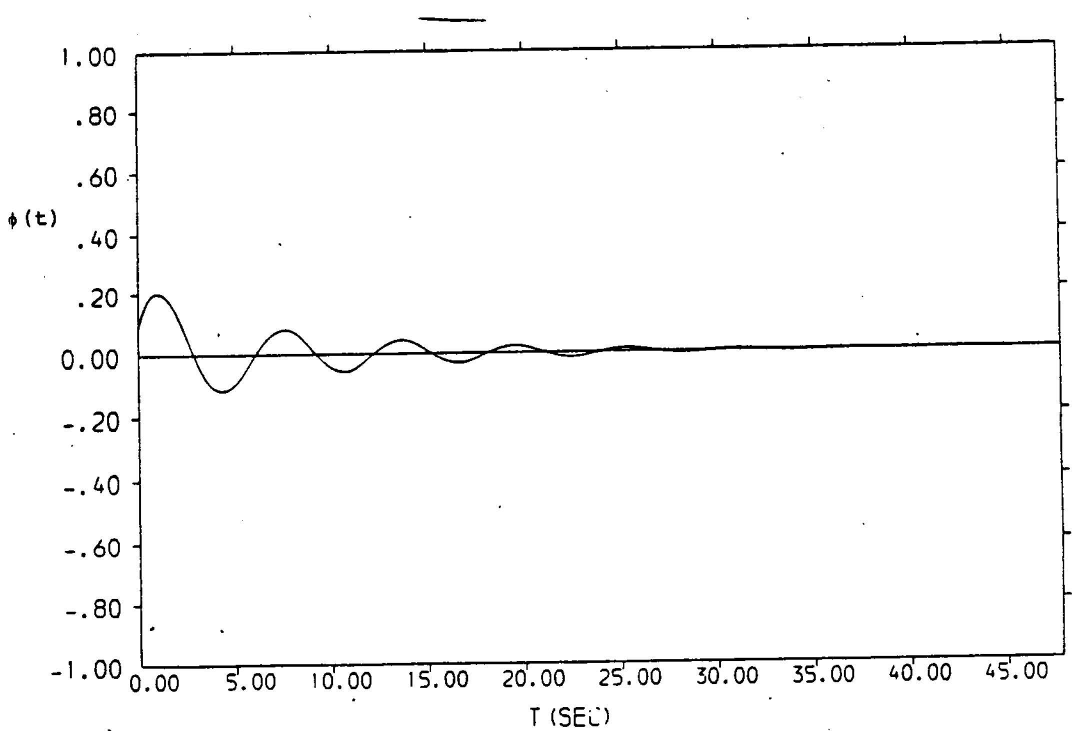


Figure 5b - Roll motion for transon stern trawler. Stable motion for exciting frequency not close to twice roll natural frequency.