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USE OF PANEL METHOD FOR SHIP HYDROSTATIC CALCULATIONS

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ABSTRACT

The panel method is a numerical technique which performs the pressure integration over the ship hull. It enables the direct calculation of the hydrostatics for any combination of waterline, heel and trim. This paper describes its development by the authors at UNO. Since the ship hull is represented by panels the panel method is more efficient than the traditional Bonjean-curves. This makes the panel method useful when calculations are made for complicated hull geometries.

Comparisons with the SHCP program results indicates good accuracy for the hydrostatic calculations of a conventional hull. The attractiveness of this method is the geometric description of the hull enables the designer to perform hydrodynamic and finite element structural calculations.

1. BASIS OF PANEL METHOD FOR HYDROSTATIC CALCULATIONS

In the panel method a direct pressure evaluation of the hydrostatic pressure is used. The outer surface of the hull is discretized into flat panel elements (see van Santen [1986] and Schalck & Baatrup [1991]). Panels of arbitrary shape are possible. The panel method is based on integration around the contour of each panel. In this manner Green's formula is utilized to convert double integrals into line integrals.

The hull is subdivided in a certain number of compartments. Each compartment is assigned its own permeability and individual compartment flooding can be simulated.

The outer surface of each compartment modeled by a certain number of flat panel elements. The panel can have any polygonal geometry.

The integrals concerning the hydrostatic characteristics which depend on the waterplane are then evaluated over the projections of the submerged parts of the panels on the water free surface.

The integrals concerning the hydrostatic characteristics which depend on underwater volume are carried over the submerged parts of all panels.

In the panel method any waterline can be specified by defining the ship draft, heel and trim.

To obtain the vessel orientation a linearized stepwise procedure is developed to search iteratively for the equilibrium waterline of the ship at a given displacement and center of gravity.

The panel size has no infuence in the calculation procedures. The only constraint regarding the panels size or shape is how well they can approximate a given geometry. With the equilibrium draft calculation there is no need to worry about the hull waterlines when defining the panels. The panel method procedure will automatically "cut" the panels along the waterline, and defines the submerged panels.

1.1 Calculation Formulation: Characteristics Related to the Waterplane+

The following characteristics related to the waterplane are defined:

(1) Waterplane Area.

$$A_W = \sum_{\substack{\text{Each}\\ \text{Comparment}\\ i}} \sum_{\substack{\text{Each}\\ \text{Panel}\\ j}} \int_{\substack{\text{Projected}\\ \text{Panel on the}\\ \text{Free Surface}}} dA$$

Where dA is an element of area.

Figure 1 shows the variables related to the projections of a panel onto the free surface.

The waterplane area can be expressed by:

$$A_W = \sum_{\substack{\text{Each}\\ \text{Comparment}\\ i}} \sum_{\substack{\text{Each}\\ \text{Panel}\\ j}} \int_{\substack{\text{Each Projected}\\ \text{Panel Contour}\\ \text{(excluding segments}\\ \text{parallel to y)}}} ydz =$$

$$= \sum_i \sum_j \sum_{\substack{\text{Each Segment}\\ \text{of the Panel}\\ \text{Contour (excluding}\\ \text{segments parallel to y)}}} (\propto x + \beta)_k \, dx$$

The above formulas were obtained by the application of Green's formula. By integration:

$$A_W = \left[\sum_i \sum_j \sum_k \left(\frac{\propto x^2}{2} + \beta x \right) \right]_{x_k}^{x_{k+1}}$$

(2) Waterplane Area Moment around x.

A similar procedure is used for calculating the waterplane area moment, etc.

The final numerical expressions are given below:

$$S_x = \left[\sum_i \sum_j \sum_k \frac{1}{2} \left(\frac{\propto^2 x^3}{3} + \propto \beta x^2 + \beta^2 x \right) \right]_{x_k}^{x_{k+1}}$$

(3) Waterplane area moment around y.

$$S_y = \left[\sum_i \sum_j \sum_k \left(\frac{\propto x^3}{3} + \frac{\beta x^2}{2} \right) \right]_{x_k}^{x_{k+1}}$$

(4) Moment of intertia of the waterplane area around x.

$$I_{xx} = \left[\sum_i \sum_j \sum_k \frac{1}{3} \left(\frac{\propto^3 x^4}{4} + \propto^2 \beta x^3 + \frac{3}{2} \propto \beta^2 x^2 + \beta^3 x \right) \right]_{x_k}^{x_{k+1}}$$

(5) Moment of inertia of the waterplane area around y.

$$I_{yy} = \left[\sum_i \sum_j \sum_k \left(\frac{\propto x^4}{4} + \frac{\beta x^3}{3} \right) \right]_{x_k}^{x_{k+1}}$$

(6) Product of inertia of the waterplane.

$$I_{xy} = \left[\sum_i \sum_j \sum_k \frac{1}{2} \left(\frac{\propto^2 x^4}{4} + \frac{2}{3} \propto \beta x^3 + \frac{\beta^2 x^2}{2} \right) \right]_{x_k}^{x_{k+1}}$$

1.2 Calculation Formulation: Characteristics Related to the Submerged Volume

As for the characteristics related to the submerged volume, lets us first consider the following variables:

- (x, y, z): global coordinates (z is a vertical axis)
- (x_p, y_p, z_p): local panel coordinates (without loss of generality, x_p is made always zero)
- (x_o, y_o, z_o): origin of the (x_p, y_p, z_p) system referred to the (x, y, z) system
- $\vec{n}$: normal to each panel
- (n_x, n_y, n_z): components of $\vec{n}$ in the (x, y, z) system
- ρ: density of water
- g: gravity

Figure 2 shows some variables related to a panel as viewed on its own plane. The following characteristics related to the submerged volume are evaluated by the panel method.

(1) Force on the vessel due to hydrostatic pressure.

$$\vec{F} = \sum_{\substack{\text{Each}\\\text{Compartment}\\ i}} \sum_{\substack{\text{Each}\\\text{Panel}\\ j}} \vec{n} \int_{\substack{\text{Submerged}\\\text{Part of the}\\\text{Panel}}} \rho g z \, dA =$$

$$= \sum_i \sum_j \vec{n} \int \rho g \left(z_o + \sqrt{1 - n_z^2}\, z_p\right) dA =$$

$$= \rho g \sum_i \sum_j \vec{n} \int_{\substack{\text{Each Contour}\\\text{Segment } k,\\\text{Excluding Segments}\\\text{Parallel to } z_p}} \left(z_o z_p + \sqrt{1 - n_z^2}\, \frac{z_p^2}{2}\right) d z_p =$$

$$= \rho g \sum_i \sum_j \vec{n} \sum_k \Bigg[z_o \left(\frac{\propto y_p^2}{2} + \beta y_p\right) + \sqrt{\frac{1 - n_z^2}{2}} \left(\frac{\propto^2 y_p^3}{3} + \propto \beta y_p^2 + \beta^2 y_p\right) \Bigg]_{y_{pk}}^{y_{p(k+1)}}$$

And similarly, the following expressions can be obtained:

(2) Moment of the submerged volume with respect to z.

$$L_z = \sum_i \sum_j n_z \sum_k \frac{1}{2} \Bigg[z_o^2 \left(\frac{\propto y_p^2}{2} + \beta y_p\right) + z_o \sqrt{1 - n_z^2} \left(\frac{\propto^2 y_p^3}{3} + \propto \beta y_p^2 + \beta^2 y_p\right) + \frac{1}{3}(1 - n_z^2)\left(\frac{\propto^2 y_p^4}{4} + \propto^2 \beta y_p^3 + \frac{3}{2} \propto \beta^2 y_p^2 + \beta^3 y_p\right) \Bigg]_{y_{pk}}^{y_{p(k+1)}}$$

(3) Moment of the force acting on a panel, around its y_p axis.

$$M_{yp} = \rho q \sum_k \Bigg[z_o \frac{1}{2} \left(\frac{\propto^2 y_p^3}{3} + \propto \beta y_p^2 + \beta^2 y_p\right) + \sqrt{1 - n_z^2}\, \frac{1}{3} \left(\frac{\propto^3 y_p^4}{4} + \propto^2 \beta y_p^3 + \frac{3}{2} \propto \beta^2 y_p^2 + \beta^3 y_p\right) \Bigg]_{y_{pk}}^{y_{p(k+1)}}$$

(4) Moment of the force acting on a panel, around its z_p axis:

$$M_{zp} = \rho q \sum_k \Bigg[z_o \left(\frac{\propto y_p^3}{3} + \frac{\beta y_p^2}{2}\right) + \frac{\sqrt{1 - n_z^2}}{2} \left(\frac{\propto^2 y_p^4}{4} + \frac{2}{3} \propto \beta y_p^3 + \frac{\beta^2 y_p^2}{2}\right) \Bigg]_{y_{pk}}^{y_{p(k+1)}}$$

(5) Point of action for hydrostatic pressure force acting on a panel:

$$x_{p,a} = 0;$$

$$y_{p,a} = \frac{M_{zp}}{F_p};$$

$$z_{p,a} = \frac{M_{yp}}{F_p},$$

where F_p is the pressure force acting on the panel.

In global coordinates:

$$x_a = x_o \frac{n_y}{\sqrt{1 - n_z^2}} y_{p,a} \frac{n_x n_z}{\sqrt{1 - n_z^2}} z_{p,a},$$

$$y_a = y_o + \frac{n_x}{\sqrt{1 - n_z^2}} y_{p,a} - \frac{n_y n_z}{\sqrt{1 - n_z^2}} z_{p,a};$$

$$z_a = \sqrt{1 - n_z^2}\, z_{p,a}$$

(6) Momentos of submerged volume with respect to x and y.

$$M_x = \sum_i \sum_j (F_z y_a - F_y z_a);$$

$$M_y = \sum_i \sum_j (-F_z x_a + F_a z_a),$$

in which (F_x, F_y, F_z) are components of the pressure force acting on each panel in respect to the (x, y, z) system.

(7) Center of buoyancy:

$$x_c = -\frac{M_y}{F_z};$$

$$y_c = \frac{M_x}{F_z};$$

$$z_c = \frac{\rho g L_z}{F_z}$$

(8) Wetted surface:

$$S = \sum_j \sum_k \sum{}' \left(\frac{\propto y_p^2}{2} + \beta y_p \right) \Bigg|_{Y_{pk}}^{Y_{p(k+1)}}$$

2. DETERMINATION OF THE POSITION OF EQUILIBRIUM OF THE FLOATING BODY

The actual waterline of a ship with a certain mass M and center of gravity (x_g, y_g, z_g) can be searched for iteratively using a linearized stepwise procedure (see Schalck & Bastrup [1991]).

Denoting

$$\delta_1 = M - \rho \nabla$$

$$\delta_2 = M_{xg} - M_x;$$

$$\delta_3 = M_{yg} - M_y;$$

where ∇ is the displacement, and M_x and M_y are, as before, the moments of submerged volume in respect to x and y, the equilibrium conditions can be written as:

$$\delta_1 = 0; \delta_2 = 0; \delta_3 = 0.$$

The procedure is the following:

(1st) an i^{th} initial waterline is assumed, being specified by the draft T_i, trim θ_i and heel ψ_i;

(2nd) the correspondent $(\delta_1)_i$, $(\delta_2)_i$ and $(\delta_3)_i$ are evaluated;

(3rd) if $(\delta_1)_i$, $(\delta_2)_i$ and $(\delta_3)_i$ are sufficiently close to zero, the process is ended, and this waterline is considered the actual one;

(4th) if not, a new waterline is obtained using linearized expressions fo δ_1, δ_2 and δ_3, valid for waterlines sufficiently close to the i^{th} one:

$$\begin{bmatrix} \frac{\partial\delta 1}{\partial T} & \frac{\partial\delta 1}{\partial\theta} & \frac{\partial\delta 1}{\partial\psi} \\ \frac{\partial\delta 2}{\partial T} & \frac{\partial\delta 2}{\partial\theta} & \frac{\partial\delta 2}{\partial\psi} \\ \frac{\partial\delta 3}{\partial T} & \frac{\partial\delta 3}{\partial\theta} & \frac{\partial\delta 3}{\partial\psi} \end{bmatrix} \begin{bmatrix} (T_{i+1)} - T_i) \\ (\theta_{i+1} - \theta_i) \\ (\psi_{i+1} - \psi_i) \end{bmatrix} = \begin{bmatrix} -\delta_1 \\ -\delta_2 \\ -\delta_3 \end{bmatrix}$$

where T_{i+1}, θ_{i+1} and ψ_{i+1} are the new values for the draft, trim and heel, and the derivatives are given by:

$$\frac{\partial\delta 1}{\partial T} = \rho A_w;$$

$$\frac{\partial\delta 1}{\partial\theta} = -\rho S_y;$$

$$\frac{\partial\delta 1}{\partial\psi} = -\rho S_x \cos\theta;$$

$$\frac{\partial\delta 2}{\partial T} = -\rho S_y;$$

$$\frac{\partial\delta 2}{\partial\theta} = -(z_g + T) M + \rho (I_{yy} + L_z + T\nabla);$$

$$\frac{\partial\delta 2}{\partial\psi} = y_g M \sin\theta - \rho I_{xy} \cos\theta - M_y \sin\theta;$$

$$\frac{\partial\delta 3}{\partial T} = -\rho S_x;$$

$$\frac{\partial\delta 3}{\partial\theta} = -\rho I_{xy};$$

$$\frac{\partial\delta 3}{\partial\psi} = -M(-x_g \sin\theta + z_g \cos\theta + T\cos\theta) -$$

$$-\rho (I_{xx} + L_z + T\nabla) \cos\theta + M_x \sin\theta;$$

(5th) the substitutions $T_i \leftarrow T_{i+1}$, $\theta_i \leftarrow \theta_{i+1}$, $\psi \leftarrow \psi_{i+1}$, are made and the process begins again at Step 2.

3. IMPLEMENTATION OF THE PANEL METHOD ON THE COMPUTER

The program HYDROSTATICS written in PASCAL was implemented in the University of New Orleans (UNO) VAX cluster. Similar programs can be written in any other computer language and implemented using either a main frame or micro computer.

The HYDROSTATICS program was first tested for a box type vessel sketched in Figure 3.

The "vessel" is a cube of side size 2T (T=1). The hydrostatic characteristics of the vessel can be expressed by analytical formulas, as follows (for $0 \leq \theta \leq \pi/4$):

$$A_w = \frac{2T}{\cos\theta};$$

$$S_x = S_y = 0;$$

$$I_{xx} = \frac{2T^3}{3\cos^3\theta};$$

$$I_{xy} = 0;$$

$$F_z = 4T^3;$$

$$F_x = F_y = 0 ;$$

$$x_x = 0 ;$$

$$y_c = \frac{1}{2T^2} \{ 2 [\frac{1}{3} T^3 \sin\theta \cos^2\theta +$$

$$+ (T\cos\theta + \frac{1}{3}(\frac{T}{\cos\theta} - T\cos\theta)) . (\frac{T}{\cos\theta} - T\cos\theta) .] - \frac{T\sin\theta}{2}$$

$$- T^3 \sin\theta\}$$

$$z_c = - \frac{T}{6} \sin\theta\tan\theta - \frac{T}{2} \cos\theta$$

Table 1 presents a comparison of the theoretical results of the hydrostatic program using the panel method. For this calculation only eight panels were given, one fo each cube face.

The search for the actual water line was also tested for the box type vessel. A mass, M = 0.5129, and a center of gravity with coordinates ($x_g = 0; y_g = 0$ $z_g = 0.250$) were assumed. The initial waterline was:

$$T_0 = 0.0$$

$$\theta_0 = 10^0$$

$$\psi_0 = 0^0$$

4. EXAMPLE OF HYDROSTATIC CALCULATION. COMPARISON WITH RESULT FROM CLASSICAL HYDROSTATIC METHOD

The hydrostatic properties of a vessel A-1 were calculated by the panel method approach (program HYDROSTATIC) and compared to results obtained by the conventional method (program SHCP). The main characteristics of the hull are:

L (length)	60.0 m
B (beam)	5.9 m
H (draft)	3.0 m
∇ (displacement)	457 m³

The body plan of vessel A-I is shown in Figure 4. The calculation was performed with the SHCP, Ship Hull Characteristics Program, and the panel method program, HYDROSTATICS. In the panel method calculations, the representation of half-hull was done using 219 boundary points and 225 panels.

For the design draft (3.0 m), the displacement and waterplane area:

	HYDROSTATIC Panel Method	SHCP	% Diff.
∇ (Displacement)	461 m³	457 m³	0.8 %
A_w (waterplane area)	316 m²	306 m²	3.0 %

The results of the hydrostatic calculations are compared in Figures 5 and 6. Examining the results it is clear that the essentially equivalent (error being less than 2%).

5. DISCUSSION AND CONCLUSIONS

The results for the box type hull showed the hydrostatic panel method to be a reliable tool for calculating hydrostatic properties. The descripti on of the hull in terms of flat panelsenables the calculations with unusual and complicated geometries.

The results for the ship hull with the panel method gives results similar to those obtained by the classical SHCP method. For conventional hulls there are advantages to adopt the panels description, if some hydrodynamics or finite elements structural calculations are to be performed, because the same hull representation can be used.

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TABLE 1.

θ	Type of Result*	A_w	I_{xx}	I_{yy}	F_z	y_c	z_c
0^o	T	1.0000	0.0833	0.0833	0.5129	0.000	-0.250
	C	1.0000	0.0833	0.0833	0.5129	0.000	-0.250
-20^o	T	1.0642	0.1004	0.0887	"	0.025	-0.245
	C	1.0642	0.1004	0.0887	"	0.025	-0.245
-40^o	T	1.3054	0.1854	0.1088	"	0.016	-0.237
	C	1.3054	0.1854	0.1088	"	0.016	-0.236
-60^o	T	1.1547	0.1283	0.0962	"	-0.028	-0.241
	C	1.1547	0.1283	0.0962	"	-0.028	-0.241
-80^o	T	1.0154	0.0872	0.0846	"	-0.014	-0.249
	C	1.0154	0.0872	0.0846	"	-0.014	-0.249
-100^o	T	1.0154	0.0872	0.0846	"	0.014	-0.249
	C	1.0154	0.0872	0.0846	"	0.014	-0.249
-120^o	T	1.1547	0.1283	0.0962	"	0.028	-0.241
	C	1.1547	0.1283	0.0962	"	0.028	-0.241
-140^o	T	1.3054	0.1854	0.1088	"	-0.016	-0.237
	C	1.3054	0.1854	0.1088	"	-0.016	-0.236
-160^o	T	1.0642	0.1004	0.0887	"	-0.025	-0.245
	C	1.0642	0.1004	0.0887	"	-0.025	-0.245
-180^o	T	1.0000	0.0833	0.0833	"	0.000	-0.250
	C	1.0000	0.0833	0.0833	"	0.000	-0.250

* T: Theoretical; C: Program

The table below shows the iterations.

TABLE 2.

Iteration	θ^o
1	10^o
2	5.29^o
3	2.69^o
4	1.35^o

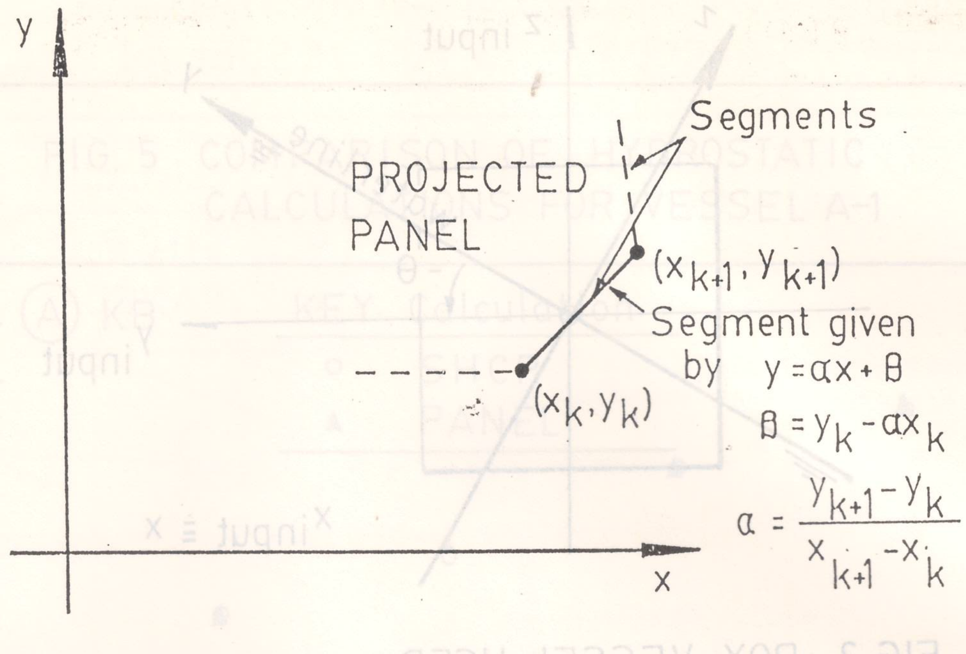

FIG. 1 PROJECTION OF PANEL ON FREE SURFACE

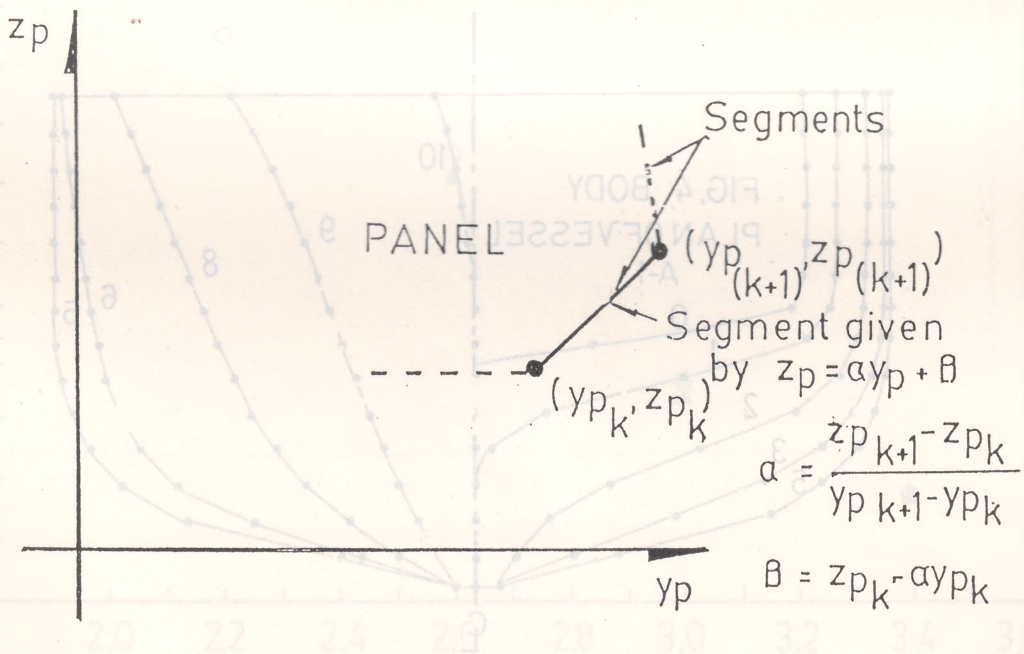

FIG. 2 PANEL IN ITS OWN PLANE

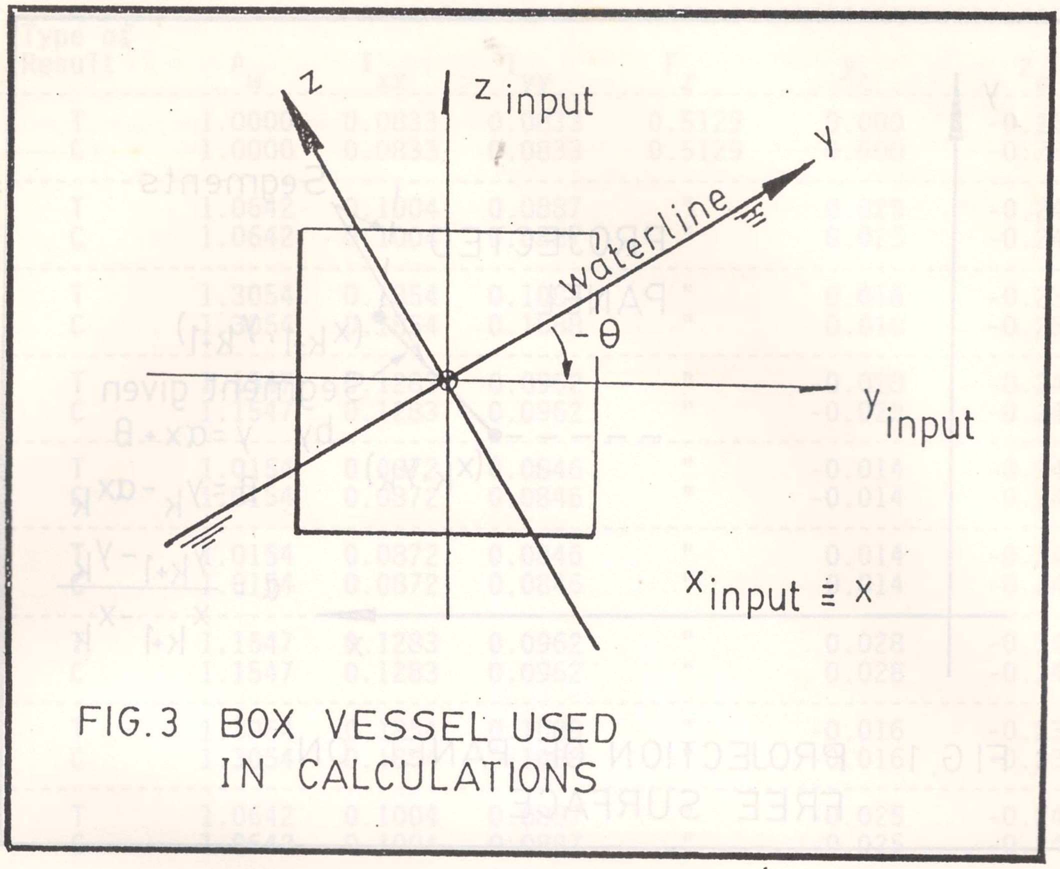

FIG.3 BOX VESSEL USED IN CALCULATIONS

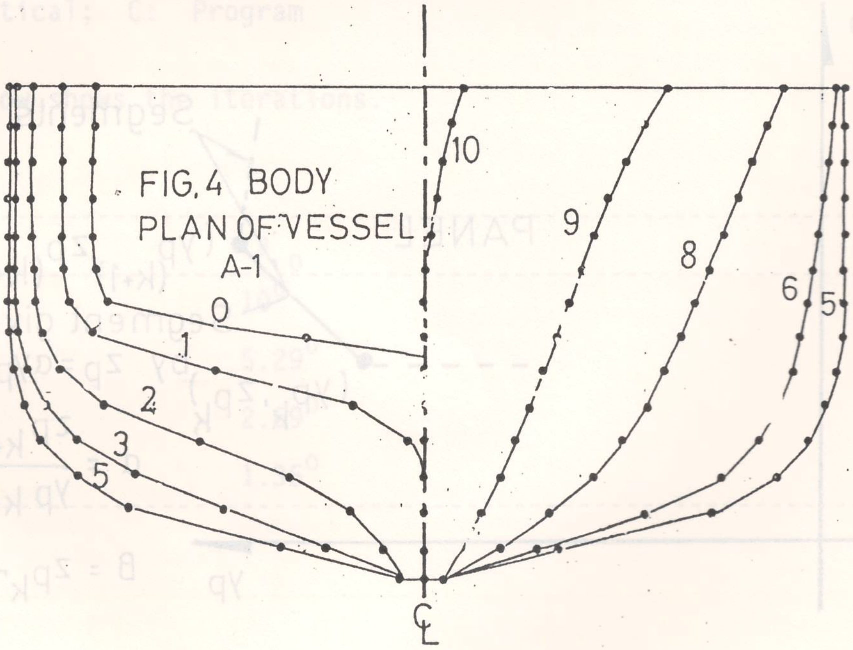

FIG.4 BODY PLAN OF VESSEL A-1

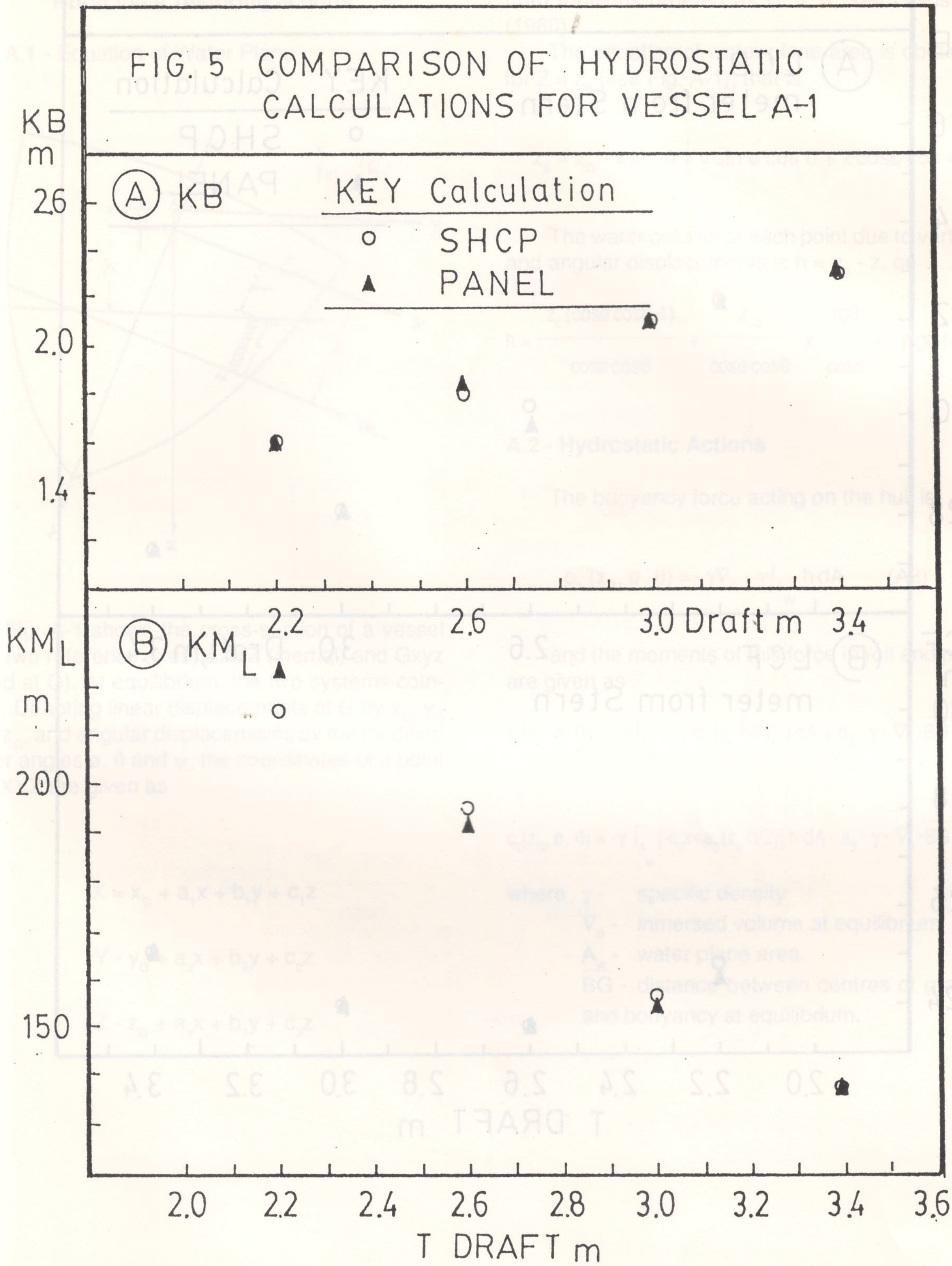

FIG. 5 COMPARISON OF HYDROSTATIC CALCULATIONS FOR VESSEL A-1

FIG. 6 COMPARISON OF HYDROSTATIC CALCULATIONS FOR VESSEL A-I