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DYNAMIC STABILITY OF FISHING VESSELS IN ASTERN SEAS

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SUMMARY

In this paper, stability of roll motion of fishing vessels in oblique waves is treated as a dynamic problem. A new non-linear mathematical model is presented, in which the six degrees of freedom of ship motions are allowed to interact, specially through non-linear hydrostatic coupling in the heave, roll and pitch modes. Results of simulations are given for a typical stern trawler. The results of simulations are given for a typical stern trawler. The results suggest that subharmonic resonances in oblique long waves may be a source of risk for some fishing vessels.

RESUMEN

En este trabajo la estabilidad del balance de buques pesqueros en olas oblicuas se trata como un problema dinámico. Un nuevo modelo matemático no-lineal se presenta, en el cual se considera interacción en los seis grados de libertad, y en especial por medio de acoplamiento hidrostático no-lineal en los modos de movimiento vertical del centro de gravedad, balance y cabeceo. Resultados de simulaciones son presentados para un arrastrero típico. Los resultados indican que resonancias sub-armónicas en olas largas y oblicuas pueden constituir una seria fuente de riesgos para algunos tipos de buques pesqueros.

1 - INTRODUCTION

A large number of fishing vessels capsizings in rough seas takes place each year (see Neves (1990)). This may be indicative that compliance with traditional stability criteria (IMO (1977)) — essentially valid for calm water conditions— do not represent an efficient safeguard for vessel integrity at sea.

Stability in waves is an aspect of fundamental importance for the safety of fishing vessels. Clearly, this is a matter of dynamics, since large motions are usually developed in a capsizing sequence. Additionally, it is well known that most ship capsizings occur when a vessel is taking waves from astern.

In this paper, stability of roll motion at a given ship speed in regular oblique waves is treated as a dynamical problem. A new mathematical model, valid for any wave incidence is presented, in which six degrees of freedom of ship motions are allowed to interact. Special attention is given to the heave, roll and pitch modes, which are coupled together by nonlinear hydrostatic terms.

The nonlinear equations of motion are then cast into a particular form that corresponds to a set of coupled Mathieu equations. It is pointed out that this set of equations displays subharmonic resonances, directly related to an internal energy transfer process, in which the energy content of different modes are chanelled onto the roll mode.

Some relevant theoretical aspects are made clear from the mathematical model, helping to get a better understanding of the capsize of vessels in astern waves. On the other hand, numerical simulations are made, using a fourth-order Runge-Kutta algorithm, in order to check the analitycal results. Results of simulations are given for a typical stern trawler. These results suggest that subharmonic resonances in oblique long waves may be a source of risk for some vessels.

2 - COORDINATE SYSTEM

Let (oxyz) be a right-handed coordinate system fixed with respect to the mean position of the ship with z vertically upward through the centre of gravity of the ship, x in the direction of forward motion, and the origin in the plane of the undisturbed free surface, as shown in Fig. 1. The ship is supposed to be rigid and to travel with a mean forward speed U at an angle χ to a train of regular waves, such that $\chi = O$ for following waves. The senses of the six-degrees-of-freedom of motion about the mean position are illustrated in Fig. 1, where q_1, q_2, and q_3 are the linear displacements surge, sway and heave, respectively; and q_4, q_5 and q_6 are the angular displacements roll, pitch and yaw, respectively.

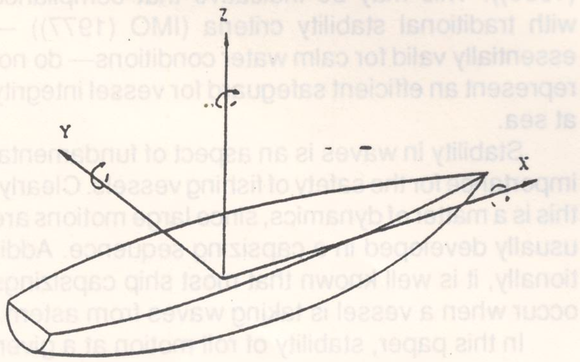

Fig. 1 - The coordinate system

3 - NON-LINEAR EQUATIONS OF MOTION

The nonlinear equations of motion may in general be expressed as

$$[M+A]\{q(t)\} + \{B(q)\} + \{C(q)\} = \{Q_e(t)\} \quad (1)$$

If consideration is given to a non-linear formulation of the roll damping due to roll motion and all other damping actions are taken as proportional to body velocities; if the restoring actions are assumed to be well behaved and are then expressed in the form of multivariable Taylor expansions about the mean position up to the 2nd. order, and considering the null coefficients in the hydrodynamic and hydrostatic terms due to geometric symmetry of the hull, then eq. (1) referred to the coordinate system described above may be written as. (Fig 1A)

The M_{ij}, A_{ij}, B_{ij}, C_{ij} elements in the first three matrices of the left hand side of equation (2) represent inertia, added mass, damping and linear restoring coefficients. The displacement and wave excitation vectors have been expressed in the form of six components:

$$q(t) = \begin{Bmatrix} x(t) \\ y(t) \\ z(t) \\ \phi(t) \\ \theta(t) \\ \psi(t) \end{Bmatrix}$$

$$Q_e(t) = \begin{Bmatrix} X_w(t) \\ Y_w(t) \\ Z_w(t) \\ K_w(t) \\ M_w(t) \\ N_w(t) \end{Bmatrix}$$

The last matrix represent non-linear restoring terms, such that a C_{ijk} element represents a second-order influence in mode i due to displacements in the j and k modes.

When non-linear restoring terms are disregarded, equation (2) is then equivalent to that given by Salvensen, Tuck and Faltinsen (1971) and Kim, Chou and Tien (1980).

Linear equations of ship motions in waves are known to offer quite good results when the interest is restricted to permanent solutions. Nowadays their use is widespread. Yet these equations do not offer any clear insight of the risks of ship capsize in waves (apart from indicating where resonance is likely to occur). As it will be made clear in the following sections, the introduction of non-linear second order restoring terms do reveal new dangerous situations for the ship in astern seas. This is, in itself, a very good reason for naval architects to face the complex field of non-linearities.

$$
\begin{bmatrix}
M+A_{11} & 0 & A_{13} & 0 & MZ_G+A_{15} & 0 \\
0 & M+A_{22} & 0 & -MZ_G+A_{24} & 0 & A_{26} \\
A_{31} & 0 & M+A_{33} & 0 & A_{35} & 0 \\
0 & -MZ_G+A_{42} & 0 & I_{xx}+A_{44} & 0 & -I_{xz}+A_{46} \\
MZ_G+A_{51} & 0 & A_{53} & 0 & I_{yy}+A_{55} & 0 \\
0 & A_{62} & 0 & -I_{xz}+A_{64} & 0 & I_{zz}+A_{66}
\end{bmatrix}
\begin{Bmatrix} \ddot{x}(t) \\ \ddot{y}(t) \\ \ddot{z}(t) \\ \ddot{\phi}(t) \\ \ddot{\theta}(t) \\ \ddot{\psi}(t) \end{Bmatrix}
+
\begin{bmatrix}
B_{11} & 0 & B_{13} & 0 & B_{15} & 0 \\
0 & B_{22} & 0 & B_{24} & 0 & B_{26} \\
B_{31} & 0 & B_{33} & 0 & B_{35} & 0 \\
0 & B_{42} & 0 & B_{44}(\dot{\phi}) & 0 & B_{46} \\
B_{51} & 0 & B_{53} & 0 & B_{55} & 0 \\
0 & B_{62} & 0 & B_{64} & 0 & B_{66}
\end{bmatrix}
\begin{Bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{z}(t) \\ \dot{\phi}(t) \\ \dot{\theta}(t) \\ \dot{\psi}(t) \end{Bmatrix}
+
$$

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & C_{33} & 0 & C_{35} & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & C_{53} & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{Bmatrix} x(t) \\ y(t) \\ z(t) \\ \phi(t) \\ \theta(t) \\ \psi(t) \end{Bmatrix}
+
$$

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \boxed{C_{333}\cdot z + C_{335}\cdot\theta} & \boxed{C_{344}\cdot\phi} & \boxed{C_{355}\cdot\theta} & 0 \\
0 & 0 & 0 & \boxed{C_{443}\cdot z + C_{445}\cdot\theta} & 0 & 0 \\
0 & 0 & \boxed{C_{533}\cdot z + C_{535}\cdot\theta} & \boxed{C_{544}\cdot\phi} & \boxed{C_{555}\cdot\theta} & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{Bmatrix} x(t) \\ y(t) \\ z(t) \\ \phi(t) \\ \theta(t) \\ \psi(t) \end{Bmatrix}
=
\begin{Bmatrix} X_w(t) \\ Y_w(t) \\ Z_w(t) \\ K_w(t) \\ M_w(t) \\ N_w(t) \end{Bmatrix}
\qquad (2)
$$

Fig 1A

4 - DETERMINATION OF COEFFICIENTS

4.1) Added mass, damping and exciting terms

Linear hydrodynamic actions for zero speed have been determined by means of a three-dimensional panel distribution method based on Green's functions, see Newman (1977), Inglis (1981), Esperança (1982). This procedure is thought to give quite accurate results for hulls with low slenderness ratios, as it is the case with typical fishing vessel hulls. For details on the panel distribution employed and hydrodynamic coefficients calculated, see Neves, Pérez and Sanguinetti (1988). Speed effects in the hydrodynamic actions are introduced following the procedure proposed by Salvensen, Tuck and Faltinsen (1970).

4.2) Roll damping

Viscous roll damping has been determined by means of the method described by Himeno (1981) in which the roll damping moment is divided into seven components, with explicit consideration of bilge keel effect. These components are due to friction, eddy-making, lift, wave-generation and bilge keel. The roll damping moment is then expressed in a quadratic form

$$B(\dot{\varnothing}) = b_1\dot{\varnothing} + b_2\,|\dot{\varnothing}|\,\dot{\varnothing}$$

Details of the procedure may be found in Pernambuco (1990).

4.3) Restoring terms

A detailled derivation of hydrostatic actions is given in Appendix A. The restoring actions in heave, roll and pitch are expanded in Taylor series, the following notation being used:

$$C_3(z,\varnothing,\theta)=C_{33}.z + C_{34}.\varnothing + C_{35}.\theta + [C_{333}.z^2 + C_{344}.\varnothing^2 + C_{355}.\theta^2] +$$

$$C_{334}.z.\varnothing + C_{335}.z.\theta + C_{345}.\varnothing.\theta$$

$$C_4(z,\varnothing,\theta)=C_{43}.z + C_{44}.\varnothing + C_{45}.\theta + [C_{433}.z^2 + C_{444}.\varnothing^2 + C_{455}.\theta^2] +$$

$$C_{434}.z.\varnothing + C_{435}.z.\theta + C_{445}.\varnothing.\theta$$

$$C_5(z,\varnothing,\theta)=C_{53}.z + C_{54}.\varnothing + C_{55}.\theta + [C_{533}.z^2 + C_{544}.\varnothing^2 + C_{555}.\theta^2] +$$

$$C_{534}.z.\varnothing + C_{535}.z.\theta + C_{545}.\varnothing.\theta$$

The linear and non-linear restoring coefficients are given in Tables 1 and 2 respectively.

TABLE 1

Linear Restoring Coefficients		
$C_{33}=\gamma.A_w$	$C_{34}=0$	$C_{35}=-\gamma.A_w.X_f$
$C_{43}=0$	$C_{44}=\gamma.\nabla.\overline{GM}_t$	$C_{45}=0$
$C_{53}=\gamma.A_w.X_f$	$C_{54}=0$	$C_{55}=-\gamma.\nabla.\overline{GM_L}$

In Tables 1 and 2 the following notation is used:

γ - specific density

∇ - volume of displacement

A_w - surface area

X_f - longitudinal position of centroid of surface area

Y_f - transversal position of centroid of surface area

Z_o - distance between origin and centre of gravity

I_x - longitudinal moment of inertia of surface area

I_y - transversal moment of inertia of surface area

GM_T - transverse metacentric height

GM_L - longitudinal metacentric height

5 - STABILITY ANALYSIS

In order to investigate the behaviour of the solutions in different conditions and to gain insight into qualitative characteristics of these solutions, it is convenient to carry out a stability analysis of the non-linear equations, prior to any exercise in numerical simulation.

TABLE 2

Second-Order Restoring Coefficients

$$C_{333} = \gamma \cdot \frac{\partial A_w}{\partial z_g} \quad ; \quad C_{334} = 0 \quad ; \quad C_{335} = -\gamma \cdot \left[-\frac{\partial A_w}{\partial \theta} + \frac{\partial (x_f A_w)}{\partial z_g} \right]$$

$$C_{343} = 0 \quad ; \quad C_{344} = -\frac{\gamma}{2} \cdot \left[z_0 \cdot A_w - 2 \frac{\partial (y_f A_w)}{\partial \phi} \right] \quad ; \quad C_{345} = 0$$

$$C_{353} = -\gamma \cdot \left[-\frac{\partial A_w}{\partial \theta} + \frac{\partial (x_f A_w)}{\partial z_g} \right] \quad ; \quad C_{354} = 0 \quad ; \quad C_{355} = -\frac{\gamma}{2} \cdot \left(z_0 A_w + 2 \frac{\partial (x_f A_w)}{\partial \theta} \right)$$

$$C_{433} = 0 \quad ; \quad C_{434} = \gamma \cdot \left[\frac{\partial (y_f A_w)}{\partial \phi} + \frac{\partial I_x}{\partial z_g} - z_0 A_w \right] \quad ; \quad C_{435} = 0$$

$$C_{443} = \gamma \cdot \left[\frac{\partial (y_f A_w)}{\partial \phi} + \frac{\partial I_x}{\partial z_g} - z_0 A_w \right] \quad ; \quad C_{444} = 0 \quad ; \quad C_{445} = \gamma \cdot \left[\frac{\partial I_x}{\partial \theta} + z_0 x_f A_w \right]$$

$$C_{453} = 0 \quad ; \quad C_{454} = \gamma \cdot \left[\frac{\partial I_x}{\partial \theta} + z_0 x_f A_w \right] \quad ; \quad C_{455} = 0$$

$$C_{533} = -\gamma \cdot \left[\frac{\partial (x_f A_w)}{\partial z_g} \right] ; \quad C_{534} = 0 \quad ; \quad C_{535} = \gamma \cdot \left[\frac{\partial I_y}{\partial z_g} - \frac{\partial (x_f A_w)}{\partial \theta} - z_0 A_w \right]$$

$$C_{543} = 0 \quad ; \quad C_{544} = \gamma \cdot z_0 \cdot x_f A_w \quad ; \quad C_{545} = 0$$

$$C_{553} = \gamma \cdot \left[\frac{\partial I_y}{\partial z_g} - \frac{\partial (x_f A_w)}{\partial \theta} - z_0 A_w \right] ; \quad C_{554} = 0 \quad ; \quad C_{555} = \gamma \cdot \left[\frac{\partial I_y}{\partial \theta} + \frac{3}{2} \cdot z_0 x_f A_w \right]$$

In Tables 1 and 2 the following notation is used :

γ - specific density

∇ - volume of displacement

A_w - surface area

x_f - longitudinal position of centroid of surface area

y_f - transversal position of centroid of surface area

z_0 - distance between origin and centre of gravity

I_x longitudinal moment of inertia of surface area

By considering a solution q(t) to be of the form

$$q(t) = \propto(t) + u(t)$$

where $\propto(t)$ - solution of the linear equation
$u(t)$ - pertubation superimposed to linear solution

the so called linear variational equation of the non-linear equation is obtained. A solution u(t)=o is considered stable when, for small initial conditions $u(t_o)$, solution $u(t) \longrightarrow 0$ when $t \longrightarrow \infty$, and is unstable otherwise, see Hahn (1967).

When this known technique is applied to equation (2), a linear equation with time dependent coefficients is obtained:

$$[M+A]\{\ddot{u}\} + [B_2(t)]\{u\} + [C]\{u\} + [D_2(t)]\{u\} = 0 \quad (3)$$

with the time-varying coefficients in $B_2(t)$ and $D_2(t)$ being derived from the non-linear terms B(ø) and D(q), respectively. Matrix $D_2(t)$ is given as

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{333}.\bar{z}+C_{335}.\bar{\theta} & C_{344}.\bar{ø} & C_{355}.\bar{\theta}+C_{335}.\bar{z} & 0 \\ 0 & 0 & C_{443}.\bar{ø} & C_{443}.\bar{z}+C_{445}.\bar{\theta} & C_{445}.\bar{ø} & 0 \\ 0 & 0 & C_{533}.\bar{z}+C_{535}.\bar{\theta} & C_{544}.\bar{\theta} & C_{535}.\bar{z}+C_{555}.\bar{\theta} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(5)

where $\bar{z}(t)$, $\bar{ø}(t)$, $\bar{\theta}(t)$ represent the steady solutions of linear equations in heave, roll and pitch, respectively. Matrix $B_2(t)$ is obtained in a similar way. Equation (3) is in the form of a set of three mutually coupled Mathieu equations in heave, roll and pitch, with the roll equation linearly coupled to the sway and yaw equations, and the heave and pitch equations linearly coupled to the surge equation.

A case of particular interest is that corresponding to $\chi = 0$ in which $ø(t) = o$. In this case the equation corresponding to the perturbed roll motion decouples from the heave and pitch equations, resulting in

$$(I_{xx}+A_{44})\ddot{u}_4 + B_{44}\dot{u}_4 + C_{44}u_4 + (C_{443},\bar{z}(t) + C_{445}.\bar{\theta}(t))u_4 = 0 \quad (6)$$

where $\bar{z}(t) = z_0\cos(Wet+\beta_3)$

$$\bar{\theta}(t) = \theta_0\cos(w_e t+\beta_5)$$

if the damping coefficient is represented as a linear function of roll angular velocity.

This is a well studied equation. It is known that this equation may be unstable for some condictions, specially near the condition

$$w_e = 2 \cdot w_4 = 2 \cdot \sqrt{\frac{\Delta \cdot GM_T}{I_{xx} + A_{44}}}$$

See Kerwin(1955), De Kat an Paulling(1989), w_4 being the roll natural frequency.

6 - STABILITY IN OBLIQUE WAVES

As shown by Neves(1981), in the case of oblique waves, there are many more conditions of possible instability when a set of Mathieu equations are coupled together, that is, whenever

$$w_e \simeq \frac{|w4 \pm wi|}{n}, \qquad i = 3, 4, 5$$

$$n = 1, 2, 3, \ldots$$

Considering n = 1 (see Hsu(1985) those conditions corresponding to very short waves are not of practical importance, since such waves do not contain much energy. So, this analysis will be restricted to three conditions, viz.

$$w_e = 2.w_4$$
$$w_e = w_5 - w_4$$
$$w_e = w_3 - w_4$$

where

$$w_3 = \sqrt{\frac{\gamma.A_w}{m + A_{33}}} \quad \text{heave natural frequency}$$

$$w_5 = \sqrt{\frac{\gamma.\nabla.\overline{GM_l}}{I_{yy} + A_{55}}} \quad \text{pitch natural frequency}$$

Therefore, if this analysis points to three conditions as potentially dangerous, simulations of the non-linear equations will be carried out near these conditions, to check on the relevance of these.

7 - NUMERICAL SIMULATION

A fourth order Runge-Kutta algorithm has been used to perform numerical integrations of eq. (2) near the resonance conditions pointed out by the stability analysis as being potentially dangerous.

A typical transom stern hull form has been chosen to numerically test the relevance of the resonant motions. This hull has been tested experimentally in longitudinal waves at zero speed of advance and showed great tendency to unstabilize under parametric resonance, as shown by Neves, Perez and Sanguinetti (1988). More details of the hull form may be found in Appendix B.

Fig. 2 shows the roll motion for the fishing vessel in a condition of parametric resonance corresponding to the Mathieu instability condition $w_e = 2.w_4$. There is a marked instability in roll in this condition. The fishing vessel is at an angle $\chi = 0$ to the waves, and consequently there is no roll exciting moment. Nevertheless the vessel rolls heavily in less than 3 cycles, due to transfer of energy from the heave and pitch modes onto the roll mode. Though this result is impressive, it does not correspond to a common situation at sea. This is due to the fact that to reach a $w_e = 2w_4$ condition in a following sea it is required that the metacentric height be quite small (such that $2.w_4$ becomes small). Even so, in the condition of Fig. 2 the waves are, "per force", quite short and steep (not common at sea).

In following seas, longer waves would induce parametric resonance at condition $w_e = w_4$. This condition is reproduced in Fig. 3, where a weak tendency to instability is found, with a small metacentric height. But in this case seven cycles are required for the roll angle to reach 30 degrees. So, it may be concluded that the $w_e = w_4$ condition is not risky for the vessel, as compared with the $w_e = 2w_4$ condition, since a slow instabilization may be easily defused, either by a change in speed (U) or in wave incidence (χ).

When the wave incidence χ is taken different from zero, the vessel is then prone to instabilities related to combinations of natural frequencies. Fig. 4 shows instability when $w_e = w_3 - w_4$ for a heading $\chi = 15°$ (oblique waves by the stern). The metacentric height was taken as $\overline{GM_T} = 0.80$ m, a quite high value even for fishing vessels, and the encounter frequency is quite low. The linear response (dotted line) shows a smooth behaviour. Yet, the non-linear response (full line), after coinciding with the linear response for about 20 seconds, unstabilizes very quickly. Figs. 5 and 6 show the heave and pitch motions at this condition. It is seen that these modes remain stable, though they were responsible for roll instability through an energy transfer process. It should be noted that the wave characteristics corresponding to the results given in Figs. 4, 5 and 6 are easily encountered at sea. This is not the case with the wave considered in Fig. 2.

The influence of obliquity of waves on instability may be appreciated in Fig. 7, where the same tuning is applied to two different headings. In the $\chi=15°$ case, instability related to the combined parametric resonance $w_e = w_3 - w_4$ is present. But when waves are aligned with the ship, combined instability is not dangerous.

11 - CONCLUSIONS

The linear equations of motions do not describe some phenomena which are crucial to a complete description of the behaviour of ships in waves. Firstly, and very important for a stability analysis near resonance conditions, viscous effects are not taken into account in a linear formulation. Secondly, the linear coupling of modes reduces the possibility of resonance to no more than three situations.

The analytical treatment presented reveals that a ship taking waves from astern may be subjected to a series of resonant conditions. Additionally, the mathematical model herein given opens the way to understand and explain how a ship may attain large angles of roll when the encounter frequency is low. This may be the case in oblique waves by the stern when $w_e = w_3 - w_4$. It is possible to find many descriptions in the literature of accidents in which large rolling developed at low frequencies in moderate following seas. See, for example, Du Cane and Goodrich (1962), Conolly (1972).

The numerical simulations indicate that some of the parametric resonances revealed by the mathematical model may be strong. On the other hand, as the parametric resonances discussed here are related to geometrical non-linearities, it is possible to relate aspects of a hull form and its tendency to roll heavily in astern seas.

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APPENDIX A

Non-Linear restoring actions

A.1 - Equation of Water Plane

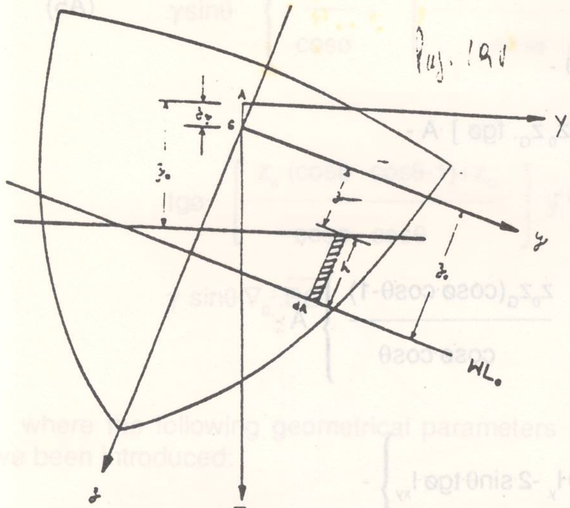

Fig. A-1 shows the cross-section of a vessel with two reference axes, AXYZ (inertial) and Gxyz (fixed at G). At equilibrium, the two systems coincide. Denoting linear displacements of G by x_G, y_G and z_G, and angular displacements by the modified Euler angles ø, θ and ψ, the coordinates of a point in AXYZ are given as

$$X = x_G + a_1x + b_1y + c_1z$$

$$Y - y_G + a_2x + b_2y + c_2z$$

$$Z - z_G + a_3x + b_3y + c_3z$$

where a_i, b_i, c_i, i = 1, 2, 3, are the directional cosines of axes Gx, Gy, Gz, respectively, obtained from rotations ordered as ψ, θ, ø, see Goldstein (1980).

The equation of water plane area is obtained for $Z = z_0$ (see Fig. A-1), that is

$$z_0 = z_G - x\cdot\sin\theta + y\cdot\sin ø\cdot\cos\theta + z\cdot\cos ø\cdot\cos\theta$$

The water column at each point due to vertical and angular displacements is $h = z_0 - z$, or

$$h = \frac{z_0(\cos ø\cdot\cos\theta - 1)}{\cos ø\cdot\cos\theta} + \frac{z_G}{\cos ø\cdot\cos\theta} - x\cdot\frac{tg\theta}{\cos ø} + y\cdot tgø \quad (A1)$$

A.2 - Hydrostatic Actions

The buoyancy force acting on the hull is

$$c_3(z_G, ø, \theta) = -\gamma\nabla_0 - \gamma\int_{A_W} h\cdot dA \qquad (A2)$$

and the moments of this force in roll and pitch are given as

$$c_4(z_G, ø, \theta) = -\gamma\int_{A_W}[c_3y - b_3(z_0 - h/2)]\,h\cdot dA + b_3\cdot\gamma\cdot\nabla_0\cdot\overline{BG} \ (A3)$$

$$c_5(z_G, ø, \theta) = -\gamma\int_{A_W}[-c_3x + a_3(z_0 - h/2)]\,h\cdot dA - a_3\cdot\gamma\cdot\nabla_0\cdot\overline{BG} \ (A4)$$

where
- γ - specific density
- ∇_0 - inmersed volume at equilibrium
- A_w - water plane area
- $\overline{BG}$ - distance between centres of gravity and bouyancy at equilibrium.

After substitution of (A1) in equations (A2), (A3) and (A4), the following hydrostatic actions, valid for large displacements, are obtained:

$$c_3(z_G, \varnothing, \theta) = -\gamma\nabla_0-\gamma\left[\frac{z_0\,(\cos\varnothing\,\cos\theta-1)A}{\cos\varnothing\,\cos\theta} \qquad \frac{z_G\,A}{\cos\varnothing\,\cos\theta} \qquad \frac{tg\,\theta}{\cos\theta}\,x_f A+tg\varnothing\,y_f A\right] \tag{A5}$$

$$c_4(z_G, \varnothing, \theta) = -\gamma\,[z_0\,(\cos\varnothing\cdot\cos\theta-1)+z_G]\cdot y_f\cdot A+\gamma I_{xy}\cdot\sin\theta\,-$$

$$\gamma I_x\cdot\sin\varnothing\cdot\cos\theta+\gamma\,[\,z_0^2\,(\cos\varnothing\cdot\cos\theta)\,tg\varnothing+z_0 z_G.\;tg\varnothing\,]\cdot A\,-$$

$$\gamma tg\varnothing\cdot\sin\theta\cdot z_0\cdot x_f\cdot A+\gamma\sin\varnothing\cdot tg\varnothing\cdot\cos\theta\cdot z_0\cdot y_f\cdot A\,-$$

$$\frac{\gamma tg\varnothing}{2}\cdot\left\{\left[\frac{z_0^2\,(\cos\varnothing\cdot\cos\theta-1)^2+z_G^2}{\cos\varnothing\cdot\cos\theta}\right]+2\,\frac{z_0 z_G(\cos\varnothing\cdot\cos\theta-1)}{\cos\varnothing\cdot\cos\theta}\right\}A\,-$$

$$\gamma.\,\frac{tg\varnothing}{2}\,.\left\{\sin\varnothing\cdot tg\varnothing\cdot\cos\theta\cdot I_x\,\frac{tg\theta}{\cos\varnothing}+\cdot\,\sin\theta\cdot I_y\;-2\cdot\sin\theta\cdot tg\varnothing\cdot I_{xy}\right\}-$$

$$\gamma tg\varnothing\cdot\left\{\frac{z_0\,(\cos\varnothing.\cos\theta-1)\cdot tg\theta}{\cos\varnothing}+\frac{z_G\cdot tg\theta}{\cos\varnothing}\right\}X_f\,A\,-$$

$$\gamma tg^2\varnothing\cdot\left\{z_0\,(\cos\varnothing\cdot\cos\theta-1)+z_G\right\}\cdot y_f A+\gamma\sin\varnothing\cdot\cos\theta\cdot\nabla_0\cdot\overline{BG} \tag{A6}$$

$$c_5(z_G, \varnothing, \theta) = \gamma\left[\frac{z_0^2\,(\cos\varnothing.\cos\theta-1)\;tg\theta}{\cos\varnothing}+\frac{z_0\cdot z_G\cdot tg\theta}{\cos\varnothing}\right]\cdot A\,+$$

$$\gamma\left[\frac{-z_0\sin\theta\cdot tg\theta}{\cos\varnothing}+z_0\,(\cos\varnothing\cdot\cos\theta-1)\;+\;z_G\right]\cdot X_f\,A\,+$$

$$\gamma\,[z_0\sin\theta\cdot tg\varnothing\cdot y_f A]\;+\;\gamma\,[-\sin\theta\cdot I_y+\sin\varnothing\cdot\cos\theta\cdot I_{xy}]\,-$$

$$\gamma \frac{\sin\theta}{2} \left\{ \left[-\frac{z_0(\cos\phi \cdot \cos\theta - 1) + z_G}{\cos\phi \cdot \cos\theta} \right]^2 \cdot A + \frac{tg^2\,\theta}{\cos^2\phi} \cdot I_y + tg^2\phi \cdot I_x \right\} -$$

$$\gamma \sin\theta \cdot \left\{ -\frac{tg\theta}{\cos\phi} \cdot \left[\frac{z_0\,(\cos\phi \cdot \cos\theta - 1) + Z_G}{\cos\phi \cdot \cos\theta} \right] \cdot x_f A + \right.$$

$$\left. tg\phi \cdot \left[\frac{z_o\,(\cos\phi \cdot \cos\theta - 1) + z_G}{\cos\phi \cdot \cos\theta} \right] \cdot y_f\, A - \frac{tg\phi \cdot tg\,\theta}{\cos\theta} \cdot I_{xy} \right\} +$$

$$\gamma\, \sin\theta \cdot \nabla_0 \cdot \overline{BG} \qquad \text{(A7)}$$

where the following geometrical parameters have been introduced:

$A\,(z_G, \phi, \theta)$ - water plane area at (z_G, ϕ, θ)
$x_f\,(z_G, \phi, \theta)$ - longitudinal position of centroid of $A(z_G, \phi, \theta)$
$y_f\,(z_G, \phi, \theta)$ - tranversal position of centroid of $A(Z_G, \phi, \theta)$
$I_x\,(z_G, \phi, \theta)$ - longitudinal second moment of $A(z_G, \phi, \theta)$
$I_y\,(z_G, \phi, \theta)$ - transversal second moment of $A(z_G, \phi, \theta)$
$I_{xy}\,(z_G, \phi, \theta)$ - product of inertia of $A(z_G, \phi, \theta)$

A.3 - Hydrostatic Coefficients

The expressions for the hydrostatic actions given above (equations (A5), (A6) and (A7)) may be expanded in Taylor series up to the second order. Expression (4) in the main text is obtained, where, in accordance with the nomenclature introduced:

$$c_{ij} = \left.\frac{\partial c_i}{\partial n_j}\right|_{o} \qquad \text{(A8)}$$

$$c_{ijk} = \left.\frac{\partial^2 c_i}{\partial n_i\, \partial n_k}\right|_{o} \quad \text{if } j \neq k \qquad \text{(A9)}$$

$$c_{ill} = \frac{1}{2} \cdot \left.\frac{\partial^2 c_i}{\partial n^2 l}\right|_{o} \qquad \text{(A10)}$$

for i, j, k, 1 = 3, 4, 5, where $n_3 = z_G$; $n_4 = \phi$; $n_5 = \theta$.

By straightforward derivation, linear hydrostatic coefficients are obtained as indicated by expression (A8); these are given in Table 1 in the main text. Deriving twice equations (A5), (A6) and (A7), non-linear hydrostatic coefficients are obtained as indicated by expressions (A9) and (A10). The non-linear hydrostatic coefficients are given in Table 2 in the main text.

APPENDIX B

Particulars of vessel

The main characteristics of the transom stern trawler used in the numerical simulations are listed below.

L_{OA}	(m)	length overall	25.91
L_{PP}	(m)	length between perpendiculars	22.09
B	(m)	beam	6.86
D	(m)	depth	3.35
T	(m)	draught	2.48
Δ	(ton)	displacement	170.30

A_w	(m^2)	water plane area	121.00
x_f	(m)	longitudinal centroid of A_w	-0,68
k_x	(m)	transversal radius of gyration	2.68
k_y	(m)	longitudinal radius of gyration	5.52
l_{bk}	(m)	length of bilge keel	$0.3L_{PP}$ to $0.8L_{PP}$
b_{bk}	(m)	width of bilge keel	0.15

The general arrangement and lay-out of the fishing vessel are given below.